

Experimental investigation of the soil-water retention of a glacial till from northern Quebec

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ABSTRACT

The soil-water retention curve represents the relationship between the pore-water deficiency and the hydraulic state of soils. Studying the SWRCs in the unsaturated state is required since they are linked to the analysis of seepage, shear strength, volume change, etc. This paper investigated the hydraulic retention behaviour of a non-plastic glacial till from northern Quebec. Different scenarios have been tested using a modified Tempe cell at Université de Sherbrooke to assess the hydraulic retention behaviour of the glacial till. It was shown that the specimen preparation techniques, the initial state and compaction efforts could all have an impact on the water storage capacity induced by suction. Moreover, using a void ratio-dependent soil-water retention model, the experimental data were modelled which can be subsequently used to study hydro-mechanical behaviour of soils in the unsaturated state.

RÉSUMÉ

La courbe de rétention d'eau du sol représente la relation entre la pression interstitielle et l'état hydraulique d'un sol. L'étude des courbes de rétention d'eau du sol est essentielle à l'état non saturé, car elle est liée à l'analyse des infiltrations, de la résistance au cisaillement, des changements de volume, etc. Cet article a examiné le comportement de rétention hydraulique d'un till glaciaire non plastique du nord du Québec. Plusieurs scénarios ont été étudiés en utilisant une cellule Tempe modifiée à l'Université de Sherbrooke pour évaluer le comportement de rétention hydraulique du till glaciaire. Il a été démontré que les méthodes de préparation des échantillons, l'état initial et les efforts de compactage peuvent influer la capacité de rétention d'eau induite par la succion. En outre, à l'aide d'un modèle de rétention d'eau dépendant de l'indice des vides, les données expérimentales ont été modélisées, ce qui peut être utilisé ultérieurement pour analyser le comportement hydromécanique du sol à l'état non saturé.

1 INTRODUCTION

Granular soils are extensively used in practice such as the filling for the soil embankments or sub-grade material for shallow foundations and road pavements. For the determination of the seepage flow through compacted granular deposits and for understanding the effect of suction on the shearing behaviour of this type of material, evaluating the soil-water retention curve (SWRC) is of particular interest. The hydro-mechanical behaviour of granular soils depends on the soil structure, which in turn is strongly influenced by the initial water content and the dry density. Thus, it can be presumed that the SWRCs are inherently dependent on the soil fabric, particle arrangement and void ratio changes in a general sense. As stated by Sun et al. (2006), the void ratio and stress history are among the dominant factors affecting the SWRC. Therefore, it is vital to investigate how the initial void ratio variation can affect the hydraulic retention.

There are two physical mechanisms responsible for the hydraulic retention behaviour of unsaturated soils,

i.e., capillarity and surface adsorption. Capillarity mechanism resulting from the presence of soil-air-water interface depends on the available volume of pores which in turn is related to the particle granulometry and the state of compaction of the soil fabric. Yet., the surface adsorption mechanism is associated with the presence of exchangeable cation hydration, mineral surface, or crystal interlayer surface hydration according to Lu and Khorshidi (2015).

Measuring the SWRC in laboratories can be influenced by many factors such as the equipment precision, sampling techniques and operational limitations. For instance, in the case of granular samples that can be easily disturbed, the particle arrangement of specimens prepared for hydraulic retention tests does not often resemble the natural soil fabric. As a result, the obtained SWRC for granular soils may not essentially represent the true hydraulic state of natural soils.

Besides, hydraulic retention tests are often timeconsuming and expensive. Hence, there is always the tendency to predict the SWRC over a given range of suction using soil-water retention models. Modelling the SWRC is also a challenging research topic since it requires a sound understanding of hydraulic retention. The empirical models, conveniently used in practice, are mathematical shape-fitting equations that may not truly represent the soil-water interactions. These models such as those proposed by Gardner (1958), Brooks and Corey (1964) and van Genuchten (1980) need the introduction of the fitting parameters to simulate the experimental results. Yet, these models do not consider the initial state of unsaturated soil as input by default. Many researchers such as Gallipoli et al. (2003), Parent et al. (2007), Nuth and Laloui (2008), Tarantino (2009), Salagar et al. (2010), and Pasha et al. (2017) attempted to relate the void ratio-dependency of SWRC to initial void ratio by implementing additional fitting parameters to the empirical models. Other than not fully distinguishing the mechanisms for hydraulic retention behaviour, there are certain shortcomings to these models. For example, Gallipoli's model shifts the whole SWRC towards higher matric potential while there are some experimental observations where void-ratio dependency is only related to the capillary domain of hydraulic retention as suggested by Tarantino (2009). The water retention behaviour at high suctions is independent of void ratio changes since the adsorbed water to the soil particles resulted from the inter-particular force interactions, notably van der Waals attraction, electric double-layer interactions do not depend on the bulk volume of pores.

The main objective of this study is to experimentally investigate how specimen preparation techniques, the initial state and compaction efforts can affect the hydraulic retention behaviour. Furthermore, the application of a void ratio-dependent soil-water retention model proposed by Maleksaeedi et al. (2019) on modelling the obtained experimental data is highlighted.

2 VOID RATIO-DEPENDENT HYBRID SOIL-WATER RETENTION MODEL

Maleksaeedi et al. (2019) presented a void ratiodependent hybrid soil-water retention model that can distinguish the mechanisms responsible for the water retention in the pores, i.e. the capillarity and adsorption mechanisms. The advantage of the model is that it can capture the effect of the initial void ratio on the main drying-wetting paths over the complete range of matric potential.

The main assumption of the model is that the void ratio-dependency of SWRC is associated with the variation of the capillarity and the adsorption component is not affected by the void ratio changes. A similar assumption has been taken by Salagar et al. (2010) and Romero and Vaunat (2000) at high suction where the water retention behaviour at suction suctions is independent of the mechanical parameters. The studies of Konrad and Lebeau (2015), Zhou et al. (2016) and Maleksaeedi and Nuth (2019) showed that the interparticle stress of particles surrounded by the liquid film is not dependent on the volume of retained liquid and is solely related to the capillary component of water retention.

The void ratio-dependent hybrid model shown in Eq. 1.

$$
\theta(\psi_m) = \theta_{cap}(\psi_m) + \theta_{ad}(\psi_m)
$$
 [1]

Where ψ_m is the matric potential, θ_{cap} is the amount of water retained in pores due to the capillarity defined by the volumetric water content. θ_{ad} is the volumetric water content corresponding to the adsorbed water.

The adsorptive component, $\theta_{ad}(\psi_m)$ describes the solid-liquid phenomenon through the Freundlich model (Jeppu and Clement 2012). Lu (2016) and Zhang and Lu (2019) suggested that θ_{ad} can be obtained using Eq. 2 as following:

$$
\theta_{ad}(\psi_m) = \theta_{max} \left\{ 1 - \left[\exp\left(\frac{\psi_m - \psi_{max}}{\psi_m}\right) \right]^M \right\}
$$
 [2]

Where θ_{max} is the adsorption capacity that ranges from 0.01 to 0.30. ψ_{max} is the highest level of matric potential that soil can undergo which can be viewed as a limiting value of pressure deficiency. M is the adsorption strength which expresses the intensity of intermolecular interaction acted on the mineral surfaces and varies from 0.01 to 1.0 as suggested by Lu (2016).

The effect of the initial state of soil on the SWRC is assumed to change in the initial void ratio influencing the capillary mechanism, that is:

$$
\theta_{cap} = f(\psi_m, e_0) \tag{3}
$$

Where e_0 is the initial void ratio during the measurement of the SWRC. In the proposed model, the capillary mechanism is described as shown in Eq. [4]:

$$
\theta_{cap}(\psi_{-}m, e_{0}) = \frac{1}{2} \left[1 - erf \left(\frac{\sqrt{2}\psi_{m}(1 - \varepsilon_{v})^{b_{1}}}{\psi_{c(ref)}} - \sqrt{2} \right) \right] \frac{\theta_{s} - \theta_{ad}(\psi_{m})}{\left[1 + (a_{ref}(1 - \varepsilon_{v})^{b_{2}}\psi_{m})^{n} \right]^{1 - 1/n}}
$$
\n[4]

In which $\psi_{c(ref)}$ and a_{ref} are the mean cavitation potential and air-entry of the reference test. b_1 and b_2 are the material's constants. ε_v is the volumetric deformation corresponding to the void ratio changes defined by Eq. 5. In this equation, $e_{0(ref)}$ is the initial void ratio of the reference test.

$$
\varepsilon_{\nu} = \frac{\Delta \nu}{\nu} = \frac{\Delta e}{1 + e_{0(ref)}}\tag{5}
$$

Warabe, et al. (2000), Kawai et al. (2000), Sun et al. (2006), and Maleksaeedi et al. [2019] indicated that any change in void ratio results in a shift in the trend of SWRC since the void ratio reduction essentially means that the volume of pores is lessened and consequently, a higher level of energy is required to empty the pores.

3 TESTING PROCEDURE AND TEST RESULTS

3.1 Soil properties

The chosen material in this study is a relatively wellgraded glacial till from northern Québec. Based on the particle size criteria set by the national building code of Canada and le ministère du transport du Québec, this type of material conforms to the granulometric requirements of MG112-type soils that are commonly used in practice as subgrade fills. The glacial till can have a minimum and maximum void ratio of 0.298 and 0.686 respectively with negligible drained cohesion and frictional angle of 37°. Figure 1 shows the compaction curve of glacial till.

The tested specimens in this study are denoted as STGi. In total, eight (8) specimens have been tested using a modified Tempe cell designed at Université de Sherbrooke (Maleksaeedi et al. 2019). The modified oedometer can track volume change and water exchange simultaneously during water retention and mechanical tests. Using this setup, the soil-water retention curves (SWRCs) under different mechanical loadings can be obtained by minimizing the friction between the loading piston and the closed oedometer cell with a low-friction O-ring. Additionally, the matric potential or suction is controlled by the axis translation or the negative water column (NWC) techniques.

Figure 1. Compaction curve of the glacial tills

3.2 Specimen preparation and test procedure

3.2.1 SWRCs with different initial water contents under constant net stress and void ratios

The moist tamping technique (MTT) was used to prepare three specimens of SGT1 to SGT3. For these specimens, a constant void ratio was targeted under different water contents. All specimens underwent constant net stress of 4 kPa during the hydraulic retention measurement.

For the tested specimens, initially oven-dried batches of glacial till were moistened by the distilled water depending on the target gravimetric water contents of 2.5%, 5.0% and 9.0%. The samples were stored in plastic bags for a day to evenly distribute the water in the soil. Afterwards, the samples were added to the modified oedometer cell and then compacted to reach the constant void ratio of 0.66 by controlling the achieved height of the specimens. The fully saturated state was achieved by injecting water to the cell and controlling the measurable parameter at hand, i.e., the injected volume of water, the dry weight of specimen and volume change measurement during the inundation process. The axis translation technique (AT) was used to impose the suction. The air-over pressure technique was employed by applying positive pore air pressure against a constant pore-water pressure of 5 kPa for all the specimens. Both volume changes and water exchanges during the experiments were recorded. The suction levels were changed only after achieving the water exchange equilibrium. After obtaining the SWRCs, the specimens were removed, and the water contents were measured for the consistency of results. It was observed the occurred errors for SGT1 to SGT3 were less than 5.0% in total.

Figure 1 illustrates the measured SWRCs of SGT1 to SGT3 in terms of the degree of saturation. As can be seen, the tested specimens had a quite similar retention capacity at lower saturation state as they merged. Given the fact that the specimens were at loose state, the main difference between the observed SWRCs was at near full saturation where SGT3 at wet-of-optimum state had a slightly higher air-entry value (AEV) compared to other specimens. The AEVs of tested specimens were in the range of 1.0 kPa at loose state.

Smith et al. (2001) stated that in general, the compaction results in the flattening of the S-shaped water retention curve. However, a clear distinction may not be discernible since changes in the water retention curve due to the compaction follows the complex relationship between the compaction process, soil properties and pore geometry. Similar behaviour was reported by Mendes and Toll (2013) on a sandy clay specimen. The SWRCs of the specimens initially prepared at different water contents were measured by the filter paper technique. It was observed that the curves converged to the primary drying SWRC at lower water content. It must be reminded that the tested specimens were essentially in a loose state and the structure of the soil has not yet changed completely due to the applied net stress compared to its natural state.

Figure 2 The SWRC of the glacial till in terms of the degree of saturation prepared by different initial water contents

3.2.2 SWRCs with different initial void ratios and water contents under constant net stress

Specimens of SGT4 and SGT5 were prepared by MTT similar to SGT1 to SGT3. Yet, for these specimens, different void ratios were achieved by compacting the specimens with different water contents. Likewise, the SWRCs were measured under constant net stress of 4 kPa by having specimens prepared at target gravimetric water contents of 2.5%, 5.0% and 9.0%. After adding samples in the modified Temp cell, the specimens were compacted by different non-standard compaction energies. The achieved dry densities were in the range of 16.5 kN/m^3 to 17.5 kN/m^3 which were lower than the dry densities obtained by the modified proctor test. After each test, the final water content was measured to control the precision of the results. A range of error between 3.0% to 7.4% was observed for tested specimens.

As can be seen from Figure 3, the effect of initial compaction and changes in the void ratio revealed itself through the variation of AEVs and the slopes of SWRCs. As the initial void ratios decreased, the AEVs increased from near 1.0 kPa to around 3.0 kPa while the slopes gradually became steeper. The compaction energy affects the relationship between the void ratio and the quantity of water retained in the pores. It seems that during the drying process, the specimens compacted near the optimum water content or at dry-of-optimum state always retained a lower amount of water under the same suction than specimens compacted at wet-ofoptimum. The findings herein are in agreement with the experimental results of Marinho and Stuermer (2000) on the residual soils obtained by the filter paper method. Based on their results, the suction capacity varies depending on the compaction energy. They stated that higher compaction energies result in a higher degree of

saturation for the same suction up to AEV. Beyond the AEV and at hydraulic residual state, the SWRCs converge to a single curve regardless of the compaction energy as observed for the glacial till in this study. They postulated that this phenomenon may be because of the pore size distribution of saturated pores is equal. They also suggested that at a very high level of suction, the electrochemical effects may be stronger than the capillary effects

Furthermore, Vanapalli et al. (1999) and Catana (2006) related the influence of compaction to the structure of unsaturated soils. They explained that the specimens with an initial water content representing the dry-of-optimum conditions have relatively large pores spaces compared to the disconnected pores on the wetof-optimum. The resulting macrostructure of specimens that are prepared at different initial water contents is different despite their identical mineralogy, texture and method of preparation.

Figure 3 The SWRC of the glacial till in terms of the degree of saturation at different initial water contents and void ratios.

3.2.3 SWRCs with different preparation techniques

To compare the effect of preparation techniques on the outcome of hydraulic retention tests, specimen SGT6 prepared by MTT and underwent a mechanical loading up to 256 kPa. The initial void ratio and water content of SGT6 were similar to SGT1.

In parallel, two specimens of SGT7 and SGT8 were assembled by the hydraulic deposition technique (HDT). To prepare the specimens, the dry batches of the glacial till were rained to the water-filled Tempe cell. After settling the soil specimen and having a clear water film on top of the specimen surface of specimens was smoothly flattened by spatula and then soil particles were allowed to deposit under their weight overnight. Afterwards, the film of water on top of each specimen was carefully removed. The specimens were vacuumed to ensure that there would not be any air bubbles within the pores. The weight and height of each specimen were then measured for the calculation phase. The advantage of the hydraulic deposition technique is that specimens have a more uniform and homogenous structure that is not manipulated by external agents like the compaction efforts. The specimens SGT7 and SGT8 underwent mechanical loading of 4 kPa and 256 kPa respectively. After performing the tests, the specimens were removed to measure the final water contents. Overall, there was a satisfactory agreement between the measured and predicted values. Figure 4 depicts the measured SWRCs of tested specimens.

Figure 4 The SWRC of the glacial till prepared by different preparation techniques in terms of the degree of saturation

From Figure 4, one can deduce that at low mechanical loading or in general terms, the confinement, specimens SGT1 and SGT7 had a similar trend. However, the difference between the SWRCs as for SGT6 and SGT8 emerged at relatively higher confinement. The AEVs of specimens at low mechanical loading were unaffected by the preparation method. Yet, at 256 kPa of mechanical loading, SGT8 had slightly higher AEV compared to SGT6. The main distinction between the SWRCs of specimens prepared with MTT and HDT at high mechanical loading is the there is a shift to a higher level of suction. Meaning that specimens prepared by hydraulic deposition had higher retention capacity at the same level of suction compared to the specimens prepared by moist tamping technique.

Furthermore, it seems that SGT8 reached the residual state at a higher saturation level compared to SGT6. The dissimilar fabrics can be the main explanation in this regard. As explored by Kumar Kodicherla et al. (2018) on ISO standard sand with fairly poor-graded grain-size distribution, hydraulic deposition techniques such as the pluviation method induce more pronounced inherent fabric anisotropy compared to moist tamping technique. As confinement increases due to the closure of pores between the particles and having a more restrained particle arrangement, the glacial till has higher retention capacity. The obtained results are consistent with numerous findings of researchers in the literature.

4 DISCUSSION ON THE SWRC MODELLING

Using Eq. 1 to 5, the SWRCs of the glacial till in section 3.2.3 were modelled. The model is only capable of modelling the main drying or wetting paths and cannot consider the looped hydraulic hysteresis. In the model, shifting the SWRC due to the initial void ratio changes is attributed to α which is commonly considered as the inverse of the air-entry value. Besides, the cavitation potential, ψ_c depends on the pore size distribution of soil and as soil structure is densified, the pore size distribution moves towards smaller radii and consequently increasing the cavitation potential.

Figure 5 Modelling the SWRCs of the glacial till with void ratio-dependent hybrid soil-water retention model

For the glacial till, as shown in Figure 5, changes in the initial void ratios along with the specimen preparation technique resulted in a shift in the trend of SWRCs particularly at relatively higher confinement of 256 kPa. However, at low confinement of 4 kPa, the variation of SWRC due to the specimen preparation technique seemed minimal. The air-entry values varied from 1.30 kPa to 4.96 kPa. These values correspond to the inverse of α in van Genuchten's model. Figure 5 also depicts the fitting parameter and the corresponding prediction of SWRC by the proposed hybrid model. To fit the experimental data, the first test with the void ratio of 0.66 was set as the reference SWRC. Afterwards, drawing the changes in the cavitation pressure and air-entry value with volumetric deformation suggested that both parameters can be fitted by a power function. As a result, by assuming θ_{max} , *M*, ψ_{max} and *n* constant, the SWRCs can be predicted in comparison to reference test results. As can be seen, the predictions are in good agreement with test data.

5 CONCLUSIONS

This paper discussed the effect of specimen preparation techniques, the initial state and compaction efforts on the retention capacity of a glacial till from northern Quebec. It was shown that at a relatively loose state for such a granular soil, regardless of the compaction efforts, specimens with a similar initial void ratio underwent a similar drying path. Yet, as the initial void ratio changes, higher compaction energies resulted in a higher degree of saturation for the same suction up to AEV. In addition, the mechanical loading can shift the SWRC to higher retention capacities, however, this increasing path might have a certain limit which is associated with the particle arrangement. Specimens prepared by the hydraulic deposition technique had higher retention capacity at higher mechanical loadings compared to the specimens prepared by the moist tamping technique which highlights the importance of soil fabric and particle arrangement on the hydraulic retention behaviour.

Using a void ratio-dependent hybrid water retention model that not only can distinguish the mechanisms responsible for the water retention in the pores but also accounts for the effect of the initial void ratio, the SWRCs were modelled. Based on the results, it was observed that the model can capture the effect of the initial void ratio on the main drying over the complete range of matric potential. This confirms the capability of the proposed model to provide an accurate estimation of hydraulic retention behaviour that ensures a better insight into the physical phenomena responsible for soilwater interaction.

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