Deriving stress/strain relationships from the contraction phase of pressuremeter tests in sands

Robert Whittle & Yasmin Byrne
Cambridge Insitu Ltd, Cambridge, England

ABSTRACT
Inserting a pressuremeter or dilatometer into sand inevitably alters the initial stress state, obscuring the early part of the test with a disturbance contribution. In extreme cases where a device has been pushed into the ground, the only part of the subsequent cavity expansion capable of straightforward analysis will be data obtained following a stress reversal. This approach is already used to derive high quality modulus measurements from unload/reload cycles (Hughes, 1982). The same argument applies to the final cavity unloading. In addition to stiffness, it is possible to derive sensible strength parameters as the unloading continues beyond the elastic range and material yields in the reverse sense. Closed form solutions that assume the form of the stress/strain response for the undrained contraction case have been widely used for some time, but for drained contraction, these give ambiguous results. This paper uses a finite difference approach based on Manassero (1989) to examine contraction data with minimal assumptions and no restrictions on the form of the stress/strain curve. Results from the unloading of self-bored, pre-bored and pushed pressuremeter tests in various dilatant materials are presented in the form of stress/strain curves with peak shear stress and friction angle identified. It is apparent that the shear strain is different for these two events, the peak shear stress occurring when the mobilized friction has reduced to the constant volume condition. Differences between the three pressuremeter insertion methods for the purposes of deriving strength are small.

RÉSUMÉ
L'insertion d'un pressiomètre ou d'un dilatomètre dans le sable modifie inévitablement l'état de contrainte initial, obscurcissant la première partie du test avec une contribution de perturbation. Dans les cas extrêmes où un appareil a été enfoncé dans le sol, la seule partie de l'expansion de la cavité ultérieure capable d'une analyse simple sera les données obtenues à la suite d'une inversion de contrainte. Cette approche est déjà utilisée pour dériver des mesures de module de haute qualité à partir de cycles de déchargement / rechargement (Hughes, 1982). Le même argument s'applique au déchargement final de la cavité. En plus de la rigidité, il est possible de dériver des paramètres de résistance sensibles au fur et à mesure que le déchargement se poursuit au-delà de la plage élastique et des rendements de matière dans le sens inverse. Les solutions sous forme fermée qui prennent la forme de la réponse contrainte / déformation pour le cas de contraction non drainée sont largement utilisées depuis un certain temps, mais pour la contraction drainée, elles donnent des résultats ambigus. Cet article utilise une approche aux différences finies basée sur Manassero (1989) pour examiner les données de contraction avec des hypothèses minimales et sans restrictions sur la forme de la courbe contrainte / déformation. Les résultats du déchargement des tests pressiométriques auto-forés, pré-forés et poussés dans divers matériaux dilatants sont présentés sous la forme de courbes de contrainte / déformation avec une contrainte de cisaillement maximale et un angle de frottement identifiés. Il est évident que la contrainte de cisaillement est différente pour ces deux événements, la contrainte de cisaillement maximale se produisant lorsque le frottement mobilisé est réduit à la condition de volume constant. Les différences entre les trois méthodes d'insertion du pressiomètre aux fins de détermination de la force sont faibles.

1 INTRODUCTION

A strain measuring pressuremeter is one of a small number of tools capable of deriving representative parameters from dilatant or granular materials. It is unique in being able derive strength and stiffness in a single test episode. The difficulty of the test is that parameters are not directly measured but are discovered by solving the boundary value problem.

There are multiple analyses available for this challenge, all of which depend on idealizing the ground response and the mechanics of the test. Given the compromises it is encouraging how close the measured response can come to the predictions of an analysis.

Figure 1 is a plot of the movements of the inside boundary of an expanding and contracting cavity in a silty dense sand. The initial cavity was made with a core barrel and the pressuremeter was placed in the cored pocket. A non-linear elastic/perfectly plastic solution adapted from Carter et al (1986) was used to find a parameter set able to account for every measured data point once it is accepted that the initial part of the loading is irrecoverable disturbance and falls outside of the solution. The details of the method are not the subject of this paper, but it is an illustration that a closed form analysis can be effective.
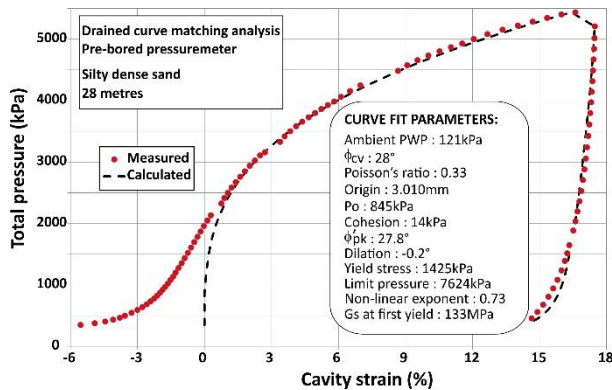

Figure 1. Matching the field curve with a parameter set

This example is a circumstance where loading and unloading data are available for analysis (the test also included small unload/reload cycles, but these have been omitted for clarity). Having valid data from both phases of the test means that parameter optimization is constrained. Although the solution is not unique the uncertainty is limited and can be quantified. The example is also a special case – the material is close to critical state. The internal angle of friction ϕ_{pk} is almost the same as ϕ_{cv} the constant volume friction angle. Consequently, there is no significant dilation and the contraction phase is straightforward to match. In general, this is not so.

This is unfortunate because whatever means is used to place the pressuremeter (self-bored, pre-bored or pushed) some insertion disturbance is inevitable. Contraction data are free of such issues. For self-bored and pre-bored tests, it is likely that some part of the loading data can be analyzed. For the pushed test the loading phase is always indeterminate, and it is only following stress reversal that the possibility for identifying realistic stress/strain properties becomes possible.

1.1   The Yielding Behaviour of Sands

The yielding response of sands under an increasing radial stress can in many cases be simplified as perfectly plastic with shear stress escalating at a constant ratio. Provided that the cavity strain remains modest (typically 10-15%) a fixed ϕ_{pk} can be assumed (Hughes et al, 1977). It is straightforward to derive the internal angle of

friction from the cavity strain and effective radial stress changes at the borehole wall as shown in Figure 2.

These assumptions do not hold when the material is unloaded. The contractive response of a sand in any condition other than critical state is not amenable to the imposition of a simple stress/strain response (for example Houlsby et al, 1986). The strain change over which the peak angle of internal friction can be sustained in contraction is small.

Manassero (1989) is a relatively straightforward numerical method intended for high quality cavity expansion tests. The advantage offered by the Manassero solution over other potentially more exact analyses such as Yu et al (1995) is the minimal number of assumptions. Very little additional information from sources external to the test are required prior to interpretation.
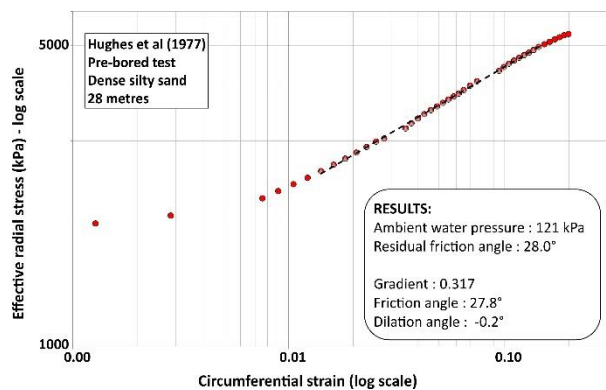

Figure 2. Finding ϕ_{pk} from cavity expansion data

It has limitations concerning the calculation of the plastic strain increments and is prone to instability if the input data are not smooth. Nevertheless, it has the potential to clarify aspects of the ground response that cannot be captured by a closed form analysis (Figure 3). Note that Figures 1-3 are all using data from the same test.
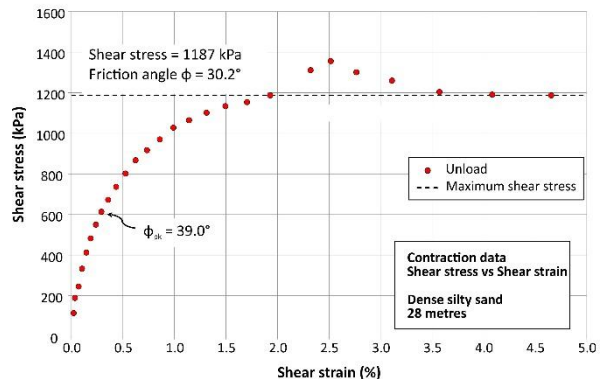

Figure 3. Applying Manassero '89 to contraction data

2 A DESCRIPTION OF THE MANASSERO 1989 SOLUTION

Solving the cylindrical cavity boundary problem means identifying the radial and circumferential strains ε_r and ε_c, and the radial and circumferential effective stresses σ'_r and σ'_c. Given any three, the fourth can be calculated. If the four parameters are known for one radius, then they are calculable for all.

At the borehole wall, ε_c and σ'_r are obtained directly from measurements the pressuremeter provides of radial displacement and total pressure P. This becomes effective stress P' when the ambient pore water pressure is deducted. The solution finds ε_r by carrying out a series of finite difference calculations using the current gradient of the measured field curve. The calculation incorporates Rowes dilatancy relationship (Rowe, 1971). This is applied as a flow rule so there is no requirement to assume deformation at a single value of friction angle. Hence the solution can be applied to tests in loose and dense sands as it permits the non-linear nature of volume change during shear.

The only additional data required that the pressuremeter test does not normally provide (but see later observations) is a value for the residual or constant volume friction angle ϕ_{cv}.

The radial strain ε_r at a point (i) corresponding to a measured data point of circumferential strain ε_c and effective pressure P' is obtained as follows:

$$\varepsilon_{r(i)} = A - B + C + D \qquad [1]$$

where:
$$A = \frac{P'(i)[\varepsilon_c(i-1)+k_a^{cv}\varepsilon_r(i-1)]}{2[P'(i)(1+k_a^{cv})-P'(i-1)]}$$

$$B = \frac{P'(i-1)\varepsilon_c(i)}{2[P'(i)(1+k_a^{cv})-P'(i-1)]}$$

$$C = \frac{P'(i)[\varepsilon_c(i-1)-\varepsilon_r(i-1)]}{2k_a^{cv}P'(i-1)}$$

$$D = \frac{P'(i-1)[\varepsilon_r(i-1)(1+k_a^{cv})-\varepsilon_c(i)]}{2k_a^{cv}P'(i-1)}$$

k_a^{cv} is the inverse of the constant volume stress ratio coefficient :

$$k_a^{cv} = \frac{1}{k_p^{cv}} = \frac{1-\sin\phi_{cv}}{1+\sin\phi_{cv}} \qquad [2]$$

The expansion of the cavity starts from zero radial strain allowing the first interval to be defined and hence starting the series of linked calculations.

Once the radial strain ε_r is known, volumetric strain ε_v and shear strain γ are calculated from the sum and difference of ε_r and ε_c. Additionally, the principal stress relationship can be obtained through:

$$\frac{\sigma_r}{\sigma_c} = k_p^{cv}\frac{\Delta\varepsilon_c}{\Delta\varepsilon_R} \qquad [3]$$

This allows the circumferential stress σ_c to be obtained and this in turn permits shear stress τ to be derived:

$$\tau = \frac{\sigma_r-\sigma_c}{2} \qquad [4]$$

The current friction angle is given by:

$$\sin\phi = \frac{\sigma_r-\sigma_c}{\sigma_r+\sigma_c} \qquad [5]$$

2.1 Application of Manassero solution to contraction data

At the end of loading, the radial stress and circumferential stress are at a maximum. For the purposes of implementing the Manassero calculations this maximum condition is treated as an origin and subsequent changes of stress and strain are calculated with respect to this point. Consequently, a plot such as Figure 3 is both an inversion and compression of the true state. This strategy is adopted to avoid having to know anything about the loading response other than the co-ordinate of the last point prior to unloading.

The modifications are straightforward. The point of maximum displacement (u_{mx}) and pressure (P_{mx}) must be identified, and an unloading pressure (P'_{adj}) and unloading cavity strain (ε_{adj}) calculated:

$$P'_{adj} = P'_{mx} - P' \qquad [6]$$

$$\varepsilon_{adj} = -\frac{u_{mx}-u}{r+u_{mx}} \qquad [7]$$

P'_{adj} and ε_{adj} replace P' and ε_c in equation 1.

Figure 4 is an example of this approach used to find the peak friction angle. Shear stress is plotted against normal stress and the slope of the initial part is used to find a value for ϕ_{pk}. These are the same data used in Figure 3 to find the internal angle of friction angle at maximum shear stress. The loading data for this test derived using Manassero is also shown in Figure 4. These data are significantly noisier for reasons discussed below. Nevertheless, the trend is similar to the unloading behaviour and supports the argument that contraction data can be used to find key soil properties independently of the loading.
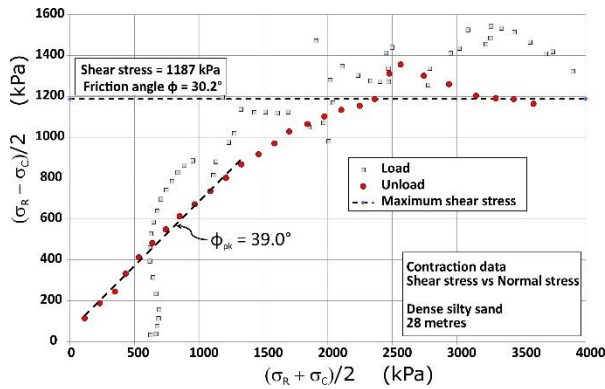
Figure 4 Shear stress vs Normal stress

Figure 5 The effects of compressibility

2.2 Issues with the Manassero method

The difficulty with implementing the analysis is that real data are generally too noisy for use as direct input. The pressuremeter data in the published paper come from ideally installed self bored tests in a chamber and are still insufficiently quiet. Manassero suggests fitting the data with a 7-degree polynomial but the form of a test is surprisingly subtle and significant detail can be masked. In particular, the necessary carrying out of unload/reload cycles causes minor disruption to the loading response but disproportionate damage to the numerical calculations (Figure 4, loading data).

Cavity unloading tends to be less affected by the disruption caused by cycling and the release of pressure is a throttled leak that gives an inherently smooth response. There is no particular difficulty is applying equation 1 directly to the measured readings. However, there is a resolution problem because the strain steps between data points are relatively coarse. This makes analysis using direct data more appropriate for strength rather than stiffness parameters.

The other potential problem is the assumption concerning strain once the material is yielding. Manassero (1989) follows Hughes et al (1977) in ignoring the elastic contribution to the plastic increment, in effect assuming that Poisson's ratio υ is 0.5. For a drained event this is not true. The error can be large, especially in less stiff materials. Figure 5 compares the expansion curves obtained with two similar analyses using the same parameter set. Carter et al (1986) includes for compressibility ($\upsilon = 0.3$). Hughes et al (1977) ignores it, as does Manassero (1989).
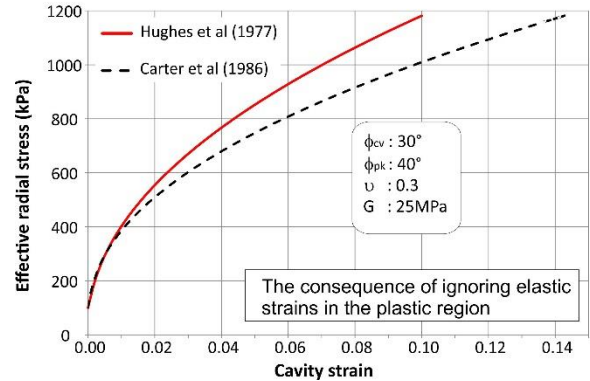
However up to about 3% shear strain the magnitude of the error is minor and the critical parts of the drained cavity contraction are within this limit. The change of stress required to achieve reverse yield typically is about 70% of the maximum pressure and the shear strain at this point will be < 2% (see Figure 1). Reverse plasticity is represented by only a few data points.

3 EXAMPLES

Figures 6 and 7 are from a test in a dense slightly silty sand that has been self-bored. There is some disturbance in the initial part of the loading. Figure 6 is an attempt to model the field curve using the same methods as were applied in Figure 1. Unlike the previous example, the sand is still dilating when the contraction phase commences.
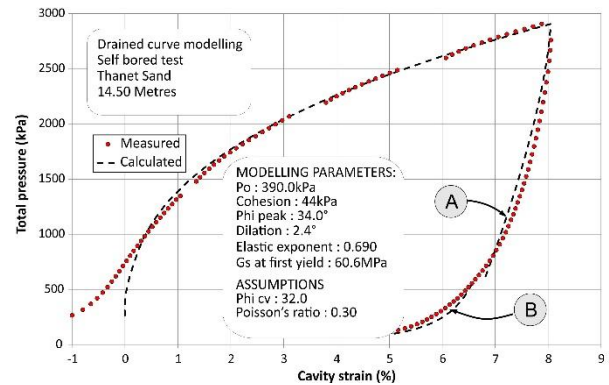

Figure 6. Curve modelling with a closed form solution

The data between the points 'A' and 'B' are not well-matched by the closed form solution. The Manassero method applied to the contraction phase of this test is shown in Figure 7, which indicates between points A and B a phase of apparently constant shear stress.
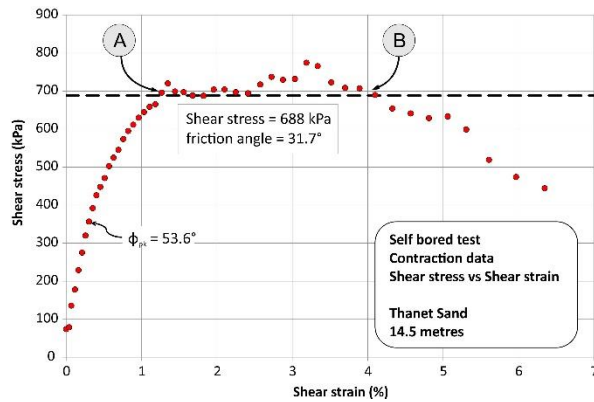
Figure 7. Contraction data plotted using Manassero 1989

Figure 9. Example of a pushed test in glacial sand

This is misleading and is a consequence of referring the stresses to a contrived origin. Point 'A' indicates where the principal stresses are equal and about to change sense – the radial stress becomes the minor stress and the circumferential or hoop stress becomes the major stress. Point 'B' marks the onset of reverse failure – it is more ambiguous than point 'A'. The relationship between the principal stresses and these points of significance are shown schematically in Figure 8 where the test starts from the Insitu lateral stress σ_{HO}.

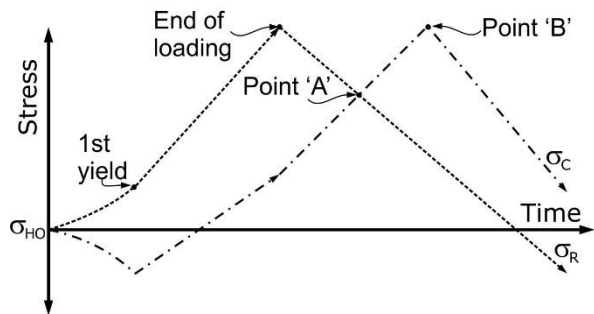Figure 10. Contraction data plotted using Manassero

Figure 8. Stress path followed during a cavity expansion and contraction

The attraction of the Manassero 1989 solution is the ability to examine the contraction phase of tests where total insertion disturbance makes the loading data indeterminate.

Figures 9 and 10 are from such a test, where a small diameter pressuremeter has been pushed into a glacial sand.
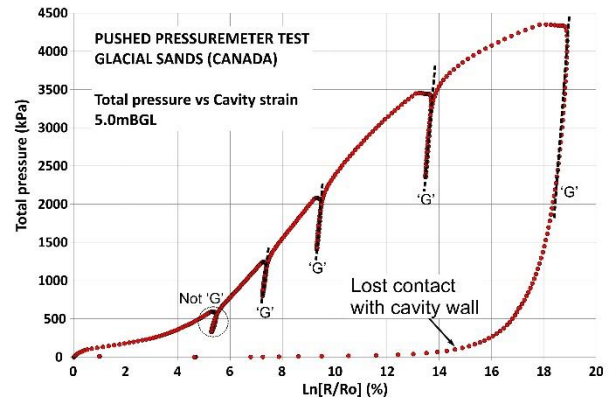
The test is shown complete, including unload/reload cycles, the first of which occurs too early in the test and does not give a representative response. Subsequent cycles are consistent and also a good match for the initial part of the final unloading.

The point where the membrane loses contact with the borehole wall is also indicated, and it is evident that the strain range for contraction is considerably smaller than that for the expansion.

The shear stress plot obtained from the contraction phase is shown in Figure 10 with a peak friction angle identified and a second friction angle taken from the point where a plateau of constant stress becomes apparent. If the assumption that this second smaller value is the constant volume angle ϕ_{cv} is correct, then it is a means through iteration of deriving this required parameter when third-party data are not available. For example, in this test the assumed value for ϕ_{cv} was 30°. The results imply that 31° would be more appropriate.

The final examples are a pair of tests taken at the same level in structureless chalk. The location was a large near-shore jack-up platform with two drill rigs set-up on opposite sides. From one rig a full displacement type pressuremeter known as an RPM was pushed down a pilot hole formed by a 10cm^2 Cone Penetrometer (CPT), whilst the other rig was used to drive a self-boring pressuremeter (SBP).
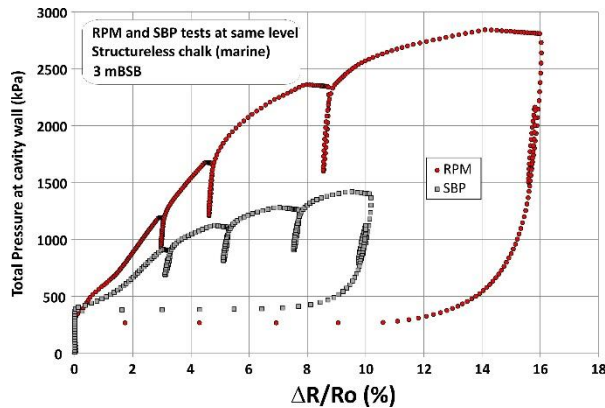
Figure 11. SBP and RPM tests in chalk

Despite an obvious difference in the loading response, the two contraction phases of the tests give almost identical results (Figure 11 and 12). The magnitude of the shear stress depends on the starting state at the point of unloading, but the peak and residual friction angles are very similar, despite the enormous difference in damage caused to the material in the vicinity of the pressuremeter by the two insertion techniques. The shear stress curve for the RPM test is noisier than the SBP test. It is a smaller diameter probe and the manner in which it is deployed means that it is vulnerable to wave action.
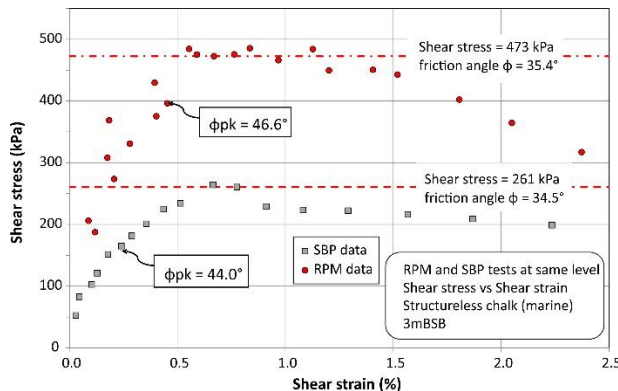

Figure 12. Contraction data for two tests in chalk

Note that the length of the strain scale is reducing for the three examples. The test in chalk (Figure 11) is only a third of the length of the silty sand (Figure 7).

4   CONCLUSIONS

There is nothing new in the analysis presented here, the novelty is the application to a part of the test that has not previously been examined in this way. Due to the special nature of the mechanics of implementing a cavity contraction (and the relative absence of creep), these data are unusually smooth and can be used as direct input to the proposed numerical solution.

The advantage of contraction data is that no matter how intrusive the insertion method used to place the pressuremeter, the unloading will give a representative response. This is well-known in the context of unload/reload cycles, and also in curve modelling *undrained* contraction (Jefferies, 1988, Whittle, 1999) but the difficulties of interpreting the drained response means that there is limited reportage of parameters from this part of the test.

Unlike the loading phase of the test where large parts are conducted at more or less the same peak friction angle, ϕ_{pk} in unloading is reducing throughout. This is complex behaviour to capture in a closed-form solution, hence the use of a numerical method.

The assumptions of the Manassero 1989 analysis are minimal and its limitations are not as significant for contraction data as they are for the loading.

The solution only considers the contribution of a purely frictional behaviour, although it has been applied to material with a modest degree of cohesion. A more complex solution that explicitly allowed for cohesion would be helpful and is an area that is currently being investigated.

Forcing the contraction data to start from zero, rather than its true state, means that what is presented is a stratagem rather than an analysis. Our suggestion is that the data are presented as plots of shear against normal stress (Figure 4), for finding the peak angle of internal angle, and plots of shear stress against shear strain (i.e Figure 7) for identifying the constant volume friction angle and significant strains.

More generally it is apparent that modelling the field response with a closed form solution that reproduces loading and contraction data is difficult. The contraction internal friction angle declines from the peak to a residual value quickly, an effect which requires a much larger strain when expanding the cavity. Capturing the response from the end of loading to yield in extension requires a non-linear function. In addition, it seems likely, from the examples shown here, that following point 'B' (Figure 8) the mobilized friction angle will remain the constant volume value.

5   CONCLUSIONS

Carter, J.P., Booker, J.R. and Yeung S.K. 1986. Cavity expansion in cohesive frictional soil, *Géotechnique*, 36(3): 349-358.

Houlsby, G.T., Clarke B.G. and Worth P.C. 1986. Analysis of the unloading of a pressuremeter test in sand, in *The Pressuremeter and its Marine Applications: Second International Symposium,* ed. J. Briaud and J. Audibert (West Conshohocken, PA: ASTM International, 1985), 245-262.

Hughes, J.M.O., Worth, C.P. and Windle, D. 1977. Pressuremeter tests in sands, *Géotechnique*, 27(4): 455-477.

Hughes, J.M.O. 1982. Determination of the Elastic Shear Modulus, ASCE *Conference on updating subsurface sampling of soil and rocks and their in situ testing*, Santa Barbra, California, USA.

Jefferies, M.G. 1988. Determination of horizontal geostatic stress in clay with self-bored pressuremeter, *Canadian Geotechnical Journal,* 25(3): 559-573.

Manassero, M. 1989. Stress-strain relationships from drained self-boring pressuremeter tests in sands *Géotechnique*, 39(2): 293-307.

Rowe, P.W. 1962. The Stress Dilatancy Relation for Static Equilibrium of an Assembly of Particles in *Contact. Proceedings of the Royal Society.* Vol. 269, Series A, 500-527.

Whittle, R.W. 1999. Using non-linear elasticity to obtain the engineering properties of clay - a new solution for the self-boring pressuremeter test, *Ground Engineering*, 32(5): 30-34.

Yu, H.S. and Houlsby, G.T. 1995. A large strain analytical solution for cavity contraction in dilatant soils, *International Journal for Numerical and Analytical Methods in Geomechanic*s, 19(11).