Waste rock disposal and segregation: Validation and upscaling of discrete element simulations

Peiyong Qiu & Thomas Pabst
Polytechnique Montréal, Montréal, Québec, Canada
Research Institute on Mines and Environment (RIME), Québec, Canada

ABSTRACT
Large volumes of waste rock are usually produced during mining operations and are often disposed of in large piles on the surface. Particle segregation caused by particle size distribution (from fine particles to meter wide blocks) and deposition methods (e.g. end dumping), usually lead to complex heterogenous internal structures, increasing the risk for instabilities. Disposal optimization could improve the geotechnical stability of piles but are difficult to test at field scale. The Particle Flow Code (PFC; Itasca) was used in this study to investigate the behavior of waste rock during disposal. Repose angle tests and segregation tests were conducted in the laboratory to calibrate and validate numerical simulations. The effect of maximum particle size was investigated. Results show that friction coefficient and rolling resistance coefficient can significantly affect the macro-properties of waste rock and are strongly related to the particle size. Relationships between numerical parameters (e.g. friction coefficient and rolling resistance coefficient) and rockfill particle size were proposed and extrapolated for field applications.

RÉSUMÉ
De grandes quantités de stériles sont généralement produites lors des opérations minières et déposées en surface dans des haldes. La distribution de la taille des particules (allant des particules fines aux blocs métriques) et les méthodes de déposition (notamment le déversement à la benne en crête), entrainent une ségrégation importante des particules et la formation de structures internes hétérogènes complexes, augmentant le risque d'instabilités. L'optimisation de la déposition pourrait contribuer à améliorer la stabilité géotechnique des haldes mais l'évaluation de ces nouvelles approches est difficile à tester à l'échelle du terrain. Le code d'écoulement de particules (PFC ; Itasca) a été utilisé dans cette étude afin d'étudier le comportement des stériles lors de leur déposition. Des essais ont été réalisés au laboratoire afin de mesurer l'angle de repos et de quantifier la ségrégation. Les résultats de ces essais ont ensuite été utilisés pour calibrer et valider des simulations numériques. L'effet de la taille maximale des particules a en particulier été étudié. Les résultats montrent que le coefficient de frottement et le coefficient de résistance au roulement peuvent affecter de manière significative les macro-propriétés des stériles et sont fortement liés à la taille des particules. Des relations entre les paramètres numériques et la taille des particules des roches stériles ont été proposées et extrapolées à l'échelle de terrain.

1 INTRODUCTION

Large volumes of waste rock are produced during mining activities, and are commonly disposed of in large piles that can exceed hundreds of meters in height and several square kilometers in area (Aubertin, 2013; Amos et al., 2015; Hajizadeh Namaghi et al., 2015). The safe management and reclamation of such large size waste rock piles represent a challenge for the mining industry.

Waste rock piles are usually constructed with benches to improve stability (Aubertin, 2013; Figure 1). Deposition by conventional construction methods (e.g. end dumping and push dumping) generally causes segregation: coarse blocks tend to move downward to the toe of the pile while fine particles remain closer to the deposition point near the crest of the pile (Nichol, 1986). The spatial variations of waste rock properties within piles can therefore present a high degree of heterogeneity (Fala et al., 2012). Fine and coarse-grained layers alternate in the pile profile as the waste rock pile grows (Anterrieu et al., 2010; Azam et al., 2007). Waste rock at the surface of each bench can also be crushed and compacted by heavy equipment during the construction, creating less permeable layers.

Heterogeneity within the pile structure can directly affect the spatial distribution of geotechnical and hydrogeological properties including porosity, density, volumetric water content, and shear strength, thus increasing the risk for localized water flow and geotechnical instabilities (Molson et al., 2005; Anterrieu et al., 2010; Lahmira et al., 2016, 2017). The optimization of waste rock disposal could result in a greater homogeneity of the structure and a better control of hydrogeotechnical properties, and therefore contribute to improve the geotechnical and hydrogeochemical stability of waste rock piles. Segregation during granular flow has been studied widely using experimental investigation and numerical simulations (Valentino et al., 2008; Utili et al., 2015; Gray, 2018). Such tests are, however, difficult to conduct and validate at field scale because of the size of the piles, the steep slopes and the large size of waste rock. A discrete element method was therefore used in this study to investigate the particle flow of waste rock blocks at laboratory and field scale.

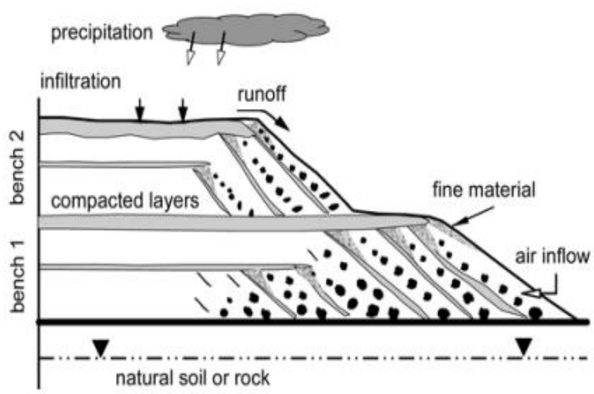

Figure 1. Conceptual cross-section of a waste rock pile (adapted from Fala et al., 2005). The external boundaries of the pile presented more visible segregation because of the less construction operation impact. The thickness and spacing between layers varied depending on the construction method, equipment types and material gradation. Water retention capacity increased as the decrease of the particle size and porosity

2 METHODOLOGY

The Discrete Element Method (DEM) can be used to simulate granular flow subjected to large deformations and free boundaries (Chung et al., 2008; Santos et al., 2016; Jiang et al., 2018; Kesseler et al., 2018). DEM has been widely used to predict the behavior of granular materials (Cundall et al, 1979; Zhu et al., 2008; Jiang et al., 2020). DEM can simulate the motion of granular material as separate particles based on Newton's second law. The deformation and energy of particles can be simulated with a force-displacement law at the inter-particle contacts (Itasca, 2020).

In this study, numerical simulations were conducted using Particle Flow Code (PFC) (Itasca, 2020), a commercial code which simulates contact interactions of granular media. DEM application requires the determination (calibration) of the microparameters (particle size, contact models and parameters) so that the flow of particles on macro-level can be representative of the reality. Laboratory or field tests are usually recommended to determine these parameters to increase calculation accuracy and reduce calculation time (Asaf et al., 2005; Franco et al., 2005; Coetzee, 2010).

Laboratory tests were thus used in this research to determine the repose angle of waste rock samples and to calibrate numerical simulations. Segregation tests were also conducted in the laboratory to calibrate and validate simulations. Waste rock for both repose angle tests and segregation tests were sampled from Canadian Malartic mine. The waste rock PSD curve (Figure 2) was established based on field measurements carried out at Canadian Malartic mine by Essayad et al. (2019), and extrapolated to maximum particle size of 1 m with combining literature (Gamache et al., 2004). The fraction with particle size ranging from 9.51 mm to 89 mm was selected to prepare the specimens for laboratory tests (blue zone in Figure 2). The effect of maximum particle size was specifically investigated by testing samples with maximum particle size between 19 mm and 38 mm. The parameters determined in the laboratory was then upscaled to simulate field conditions.

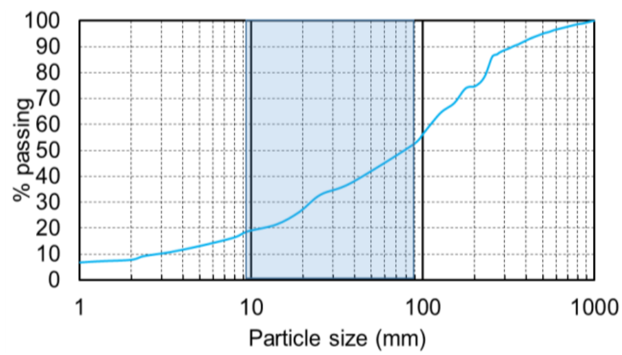

Figure 2. Field particle size distribution curve of waste rock in Canadian Malartic Mine. The section with particle size smaller than 25.4 cm was determined from field measurement at Canadian Malartic Mine (Essayad et al., 2019). The PSD curve was extrapolated to the maximum particle size of 1 m based on reported data in the literature (Gamache-Rochette, 2004).

2.1 Repose angle tests

Three particle size distribution (PSD) with different maximum particle sizes (19 mm, 25 mm, 38 mm) were used to conduct repose angle tests. The mass of the three waste rock specimens was approximately 23 kg. Repose angle tests were conducted by lifting a 30 cm high and 27 cm diameter cylinder filled with waste rock (Figure 3). Waste rock would fall down from the bucket and form a conic pile on the ground. The repose angle of the pile (θ in Figure 3) was determined from the measured pile diameter (D) and the pile height (h) with measurement precision of 0.5 cm. Each test was repeated 10 times for a maximum particle size of 19 mm, 25 mm and 38 mm.

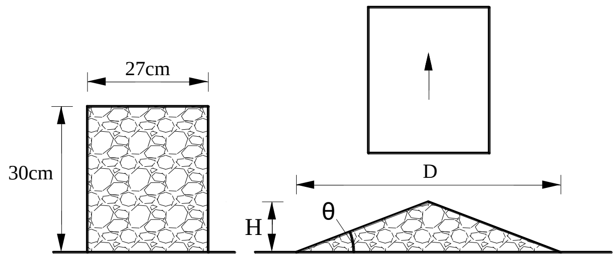

Figure 3. Repose angle test setup in the laboratory. D: the bottom diameter of the cone, H: the height of the cone.

2.2 Segregation tests

Segregation tests were setup to validate simulations at small scale. Specimens were prepared with maximum particle size of 89 mm. Ten buckets of waste rock were dumped from 1 meter high along a side wall which resulted in the formation of a waste rock slope (Figure 4). The mass of waste rock in each bucket was 22 kg. The dimensions of the buckets were 27 cm in diameter and 30 cm in height. The formed slope was then decomposed in 6 sections (from 1 to 6, see Figure 4). The PSD curves in each section were measured and compared using D_{30} and D_{50}, with a precision of 1 mm. D_{30} and D_{50} correspond to a cumulative 30% and 50% pass particle size in the particle size distribution curve. Sections 1 – 3 were mainly investigated because the segregation can be significant along the surface. Ratio hi / h of 1, 0.667, and 0.333 were used to define sections 1- 3, respectively. The slope angle in each test was also measured.

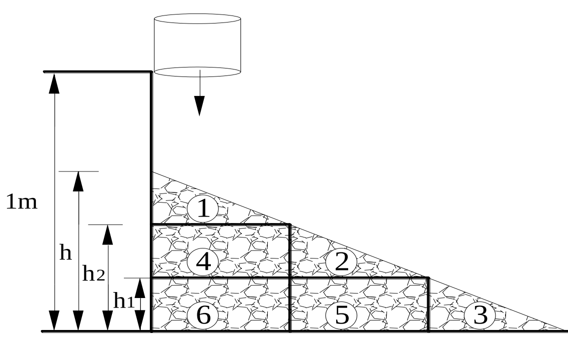

Figure 4. Segregation test setup. Each test consisted of 10 dumps and were repeated five times. Section numbers upon dismantling are shown. Each section was characterized for its PSD curve. Parameters hi and h represent the top height of the waste rock for each section and the total height of the pile, respectively.

2.3 Calibration and validation of numerical simulations

Individual elements (such as waste rock particles) are commonly simulated as spherical particles in DEM simulations. Contacts between balls are, indeed, easier to detect, thus accelerating significantly the calculations compared to more complex particle shapes (Grima et al., 2011; Su et al., 2019). Spheres are also preferred for industrial application because of the limitation of particle number and calculation capacity in the numerical codes (Marigo et al., 2015). In this study, waste rock particles were therefore simulated as spheres of various diameters. However, bulk friction is usually too small to simulate real granular material with irregular shapes (such as waste rock) when using spherical particles. Rolling resistance coefficient was introduced to consider the shape effect (Plassiard et al., 2009; Ai et al., 2011; Wensrich et al., 2012; Coetzee, 2016; Roessler et al., 2018)

The rolling resistance linear (rrlinear) model was used in the present simulations. The rolling resistance linear contact model in PFC incorporates a torque acting on the contacting pieces to counteract rolling motion (Wensrich et al., 2012). Friction coefficient (μ) and rolling resistance coefficient (μ_r) are the main parameters in controlling the flow behavior of particles (Zhou et al., 2002). Parameters μ and μ_r were therefore calibrated based on the repose angle measured in the laboratory (Figure 3). Segregation tests (Figures 4) were simulated afterwards with calibrated parameters (e.g. friction and rolling resistance coefficients) to validate the numerical simulations with rrlinear model.

The flow behavior of waste rock simulated with rrlinear contact model is also related the particle density (β), stiffness ratio (k) and Young's modulus. These parameters were determined based on lab tests and literature review. The effective modulus was set to 1×10^6 Pa to maintain the system within the rigid limit while optimizing the calculation speed (Itasca,2020). Local damp (α) was used in the simulation and set to 0.4 according to drop tests carried out in the laboratory (Figure 5). The particle density was set to 2760 kg/m^3 according to relative density tests carried out in the laboratory (Sylvain et al., 2019).

Percent error (PE) was used to evaluate the accuracy of DEM simulations from the difference between the results of simulation (R_S) and laboratory tests (R_T) (Derakhshani et al., 2015). The percent error was calculated from the following equation:

$$\mathrm{PE} = |(\mathrm{R}_S - \mathrm{R}_T)|/\mathrm{R}_T \times 100\% \qquad [1]$$

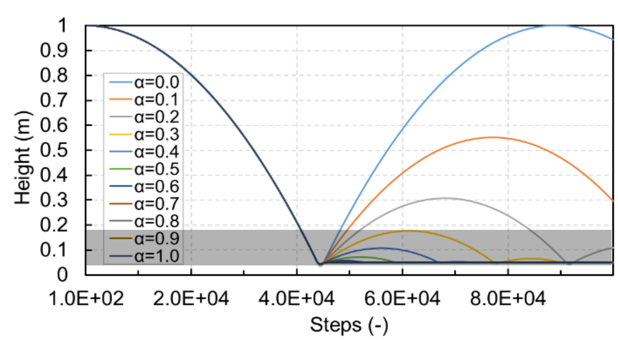

Figure 5. Determination of local damp (α). Local damping values between 0.0 and 1.0 were simulated and rebound heights were compared with drop tests conducted in the laboratory. The average rebound height of waste rock particle of 0.1 m diameter was between 0.05 m and 0.2 m, with an average close to 0.1m (shaded zone). A local damping around 0.4 seemed therefore to match the rebound height measured in the laboratory.

3 RESULTS

3.1 Repose angles

The repose angle of three specimens increased from 23.1° to 24° with maximum particle size increased from 19 mm to 38 mm (Table 1). The standard deviation of the repose angle was always lower than 1°, which indicates good reproducibility of the laboratory tests. The difference of repose angles for different maximum particle size was limited and globally less than 1° (i.e. in the same range as the precision of the test).

Table 1. Waste rock repose angle (θ) measured in the laboratory for various maximum particle size (D_{max}). Minimum, maximum and average repose angles, and standard deviation are presented. A total of 10 tests were carried out for each specimen.

D_{max} (mm)	Minimum θ (°)	Maximum θ (°)	Average θ (°)	Standard deviation (°)
19	22	23.9	23.1	0.7
25	22	24	23.4	0.7
38	22.4	24.9	24	0.9

3.2 Calibration

Simulations were conducted to reproduce the repose angle tests. Friction coefficient (μ) was varied between 0.15 and 0.4 and rolling resistance coefficient (μ_r) between 0.1 and 0.4 until the best fitting parameters (i.e. minimum PE; equation 1) were obtained to precisely simulate laboratory results. Many sets of μ and μ_r could make the simulated repose angle match the tested repose angle. Every friction coefficient μ was therefore simulated with a set of μ_r ranging in 0.1-0.4 (Figure 6). Every rolling resistance coefficient (μ_r) was also simulated with determined μ for particle size of 19 - 38 mm (Figure 7).

The repose angle tended to increase with friction coefficient for given rolling resistance coefficients. The repose angle almost showed the same as particle size increased from 19 mm to 25 mm for a given μ, but decreased as particle size increased from 25 mm to 38 mm. The μ was determined and set to 0.191, 0.195 and 0.223 for particle size of for maximum particle size 19 mm, 25 mm and 38 mm based on the fitted trend (Figure 6).

The repose angle also increased with rolling resistance coefficient in general (Figure 7). But the repose angle fluctuated when rolling resistance coefficient further increased.

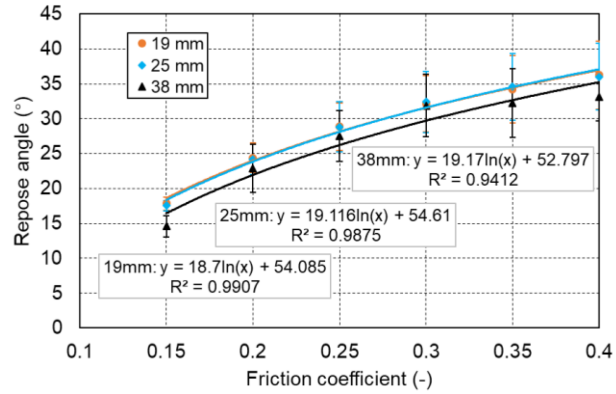

Figure 6. Variation of simulated repose angle with friction coefficient (μ) for different maximum particle sizes. Rolling resistance coefficient ranged in 0.1-0.4.

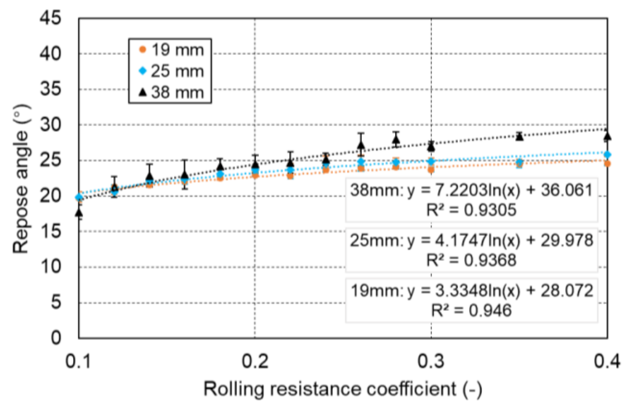

Figure 7. Variation of repose angle as a function of rolling resistance coefficient (μ_r) for different maximum particle size. Friction coefficient was set to 0.191, 0.195 and 0.223 for maximum particle size 19 mm, 25 mm and 38 mm, respectively.

The friction coefficients (μ) were calibrated to 0.191, 0.195 and 0.223, and the resistance coefficient (μ_r) to 0.225, 0.207 and 0.188 for particle size of 19 mm, 25 mm and 38 mm, respectively (Table 2). The differences between calibrated repose angles and tested repose angle were smaller than 2% (i.e. less than 1 degrees; Table 2).

Table 2 Calibrated parameters for different maximum particle sizes.

Maximum particle size	19 mm	25 mm	38 mm
Laboratory repose angle (°)	23.1	23.4	24.0
Simulated repose angle (°)	23.5	23.5	23.9
Error	1.7%	0.4%	0.4%
Friction coefficient, μ	0.191	0.195	0.223
Rolling resistance coefficient, μ_r	0.225	0.207	0.188
Local damp, α	0.4	0.4	0.4
Effective modulus, E (Pa)	1.0×10^6	1.0×10^6	1.0×10^6

3.3 Validation

Calibrated friction coefficient (μ) and rolling resistance coefficient (μ_r) for waste rock smaller than 38 mm were extrapolated for waste rock with maximum particle size of 89 mm tested in segregation tests. The friction coefficient was extrapolated and set to 0.26, while the rolling resistance coefficient was set to 0.14, based on the trend established in laboratory (Figure 8). The extrapolated values of μ and μ_r were then applied to the numerical model for segregation test (Figure 9). Laboratory and numerical results were compared based on D_{30} and D_{50} (Figure 11).

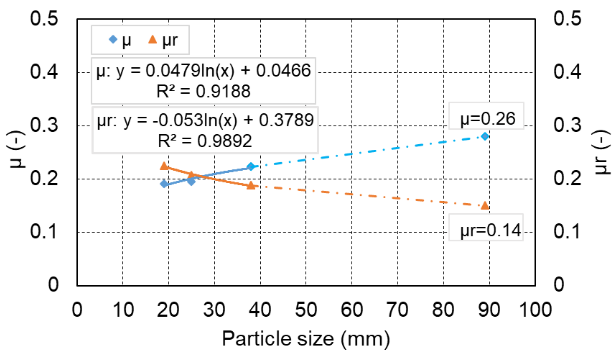

Figure 8. Relationship between calibrated rolling resistance coefficient and maximum particle size. Solid lines indicate the relation based on laboratory tests (for particles between 19 and 38 mm). Dashed lines indicate extrapolation to maximum particle size 89 mm.

D_{50} in laboratory increased by 124% from the top (S1) to the bottom (S3) layer, while D_{30} increased by 34%, indicating strong segregation from the laboratory tests.

Simulated and measured PSD curves in middle (S2) and bottom (S3) layers showed similar trends for particle size larger than 20 mm, i.e. the difference was smaller than 5% for each size in the PSD curve. Slight differences (i.e. percentage passing) were nevertheless observed for fractions smaller than 60 mm in middle layer (S2).

Simulated D_{50} from the top to the bottom of the slope increased from 23.7 mm to 50.3 mm, and matched well the D_{50} in laboratory which was comprised between 26.3 mm and 59 mm (Figure 11). D_{30} also tended to increase from the top to the bottom, both in laboratory and simulation with differences smaller than 3 mm in each layer. Both simulations and laboratory tests therefore showed marked segregation of particles along the slope. The percent error of D_{50} was 11% on the top of the pile and 17% at the bottom, and the percent error of D_{30} was 13% on the top of the pile and 7% at the bottom.

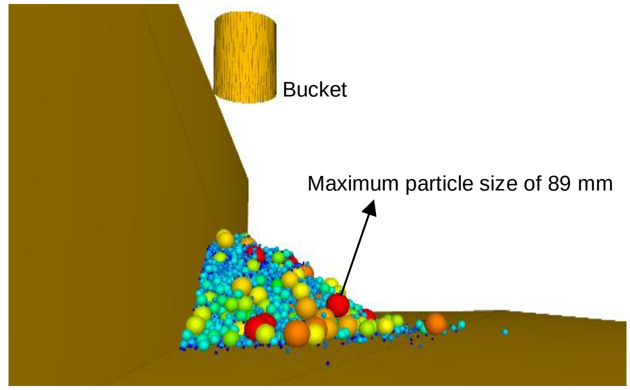

Figure 9. Segregation validation in PFC simulation. Particles are colored according to their size (from blue – small particles – to red – large particles).

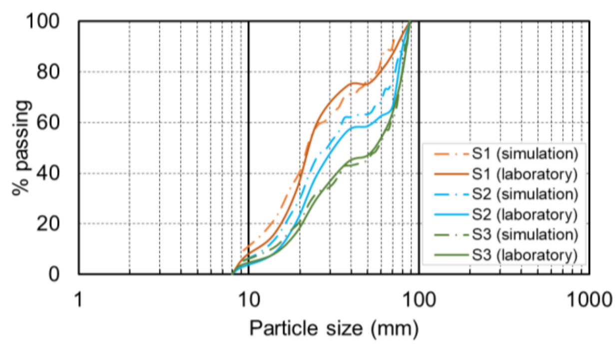

Figure 10. Measured (solid lines) and simulated (dashed lines) particle size distribution curves for sections 1-3 (S1 to S3).

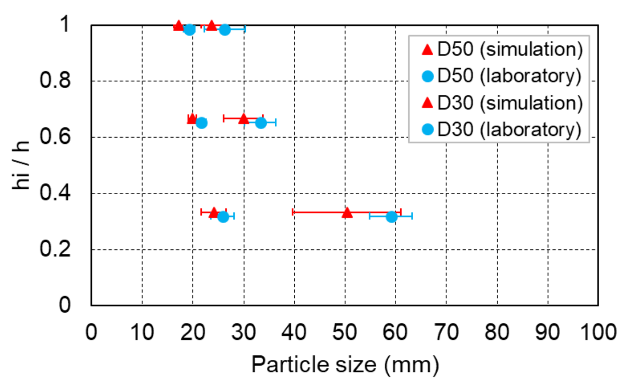

Figure 11. Measured (blue) and simulated (red) distribution of D_{50} and D_{30} along the slope in segregation tests. Ratio hi/h of 1, 0.667, 0.333 represent the elevation of the pile (top (S1), middle (S2) and bottom layer (S3) of the layer). Smaller hi/h ratio means further distance to the top of the pile.

3.4 Application to field scale

Simulation of segregation tests partly validated the extrapolation approach of friction coefficient (μ) and rolling resistance coefficient (μ_r). A field-scale model (Figure 12) was therefore simulated by extrapolating friction coefficient (μ = 0.38) and rolling resistance coefficient (μ_r = 0.01) to maximum particle size of 1 m. 45 tons of waste rock with maximum size of 1 m were dumped from 30 m high for every dump. A total of 60 dumps were simulated, corresponding to a total of 2700 tons of waste rock.

The blue color on the surface of the pile indicated that more particles with smaller sizes were accumulated on the top, while most larger sizes particles (in bright color) were accumulated on the bottom of the pile, which was consistent with field observations (Nichol, 1986). Most of the particles were accumulated on the bottom part of the slope because the amount of the particles was still small from the 60 dumps. More dumps are needed for the simulation to characterize the segregation in field.

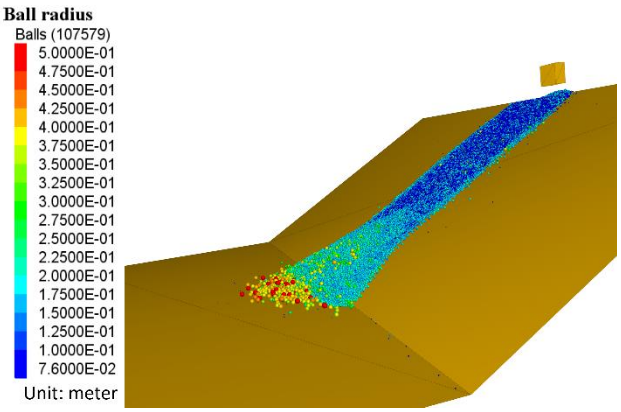

Figure 12. Simulation of waste rock dump in field scale. Particles are colored according to their size (color scale in meter).

4 RESULTS ANALYSIS AND DISCUSSION

Friction coefficient (μ) was the most critical parameter affecting the repose angle of waste rock, as observed by others for the simulations of granular materials (Just et al., 2013; Santos et al., 2016). Higher friction coefficient can tolerate a larger magnitude of the elastic deformation in the tangential direction and enhance contact stability (Zhou et al., 2002).

The repose angle increased rapidly with increasing rolling resistance coefficient. The repose angle only slightly increased with rolling resistance coefficient for low friction coefficients. Similar trend was also observed in repose angle simulations with 2 mm diameter sphere particles (Yan et al., 2015; Coetzee, 2017).

The friction coefficient increased slightly with maximum particle size (Figure 8), while the rolling resistance coefficient tended to decrease. The trend obtained from laboratory tests (Figure 8) should, however, be further evaluated for larger particles because the maximum particle size in this paper were significantly smaller than in the field.

D_{30} and D_{50} increased by 34% and 124% respectively in the toe of the pile compared to the top, thus indicated a strong segregation of waste rock particles during deposition. The simulation of segregation test using extrapolated friction coefficient and rolling resistance coefficient somewhat underestimated segregation with 41% increase for D_{30} and 113% increase for D_{50} from the top to the toe of the pile. This indicated that particles with larger size significantly affect segregation in the simulations.

Previous research indicated that many parameters, including the particle density, stiffness and effective's modulus, had a slight influence on the repose angle (Zhou et al., 2002; Derakhshani et al., 2013; Just et al., 2013). Further optimization of numerical simulations will focus on a more precise estimation of these parameters.

The repose angle measured in the laboratory was smaller than typical values of 25°-45° for gravel (Al-Hashemi, 2018), probably because of the limited container size and waste rock maximum particle size. Field-scale tests (e.g. PSDs and repose angle determination) are currently being conducted to more precisely estimate repose angle at field scale.

5 CONCLUSION

The present paper aimed to validate and upscale discrete element simulations of particle segregation during waste rock disposal. Repose angle tests were carried out in the laboratory to calibrate friction coefficient and rolling resistance coefficient in PFC simulations and segregation tests were conducted to validate the simulation models in laboratory scale. Simulations with calibrated friction coefficient and rolling resistance coefficient were able to reproduce well laboratory scale segregation tests. The extrapolation of fitted parameters from maximum size of 38 mm to samples with maximum size of 89 mm in particular seemed reasonable. Fitted parameters will be further investigated based on field investigation to simulate larger scale applications.

6 ACKNOWLEDGEMENT

The authors would like to acknowledge the financial support from NSERC, FRQNT and the industrial partners of the Research Institute on Mines and the Environment (http://rime.ca/). Thanks are also expressed to Noura El-Harrak, Patrick Bernèche, Moise Rousseau, and other colleagues in RIME for their help with the experimental tests.

7 REFERENCES

Ai, J., Chen, J.F., Rotter, J.M. and Ooi, J.Y. 2011. Assessment of rolling resistance models in discrete element simulations. *Powder Technology*, 206(3): 269-282.

Al-Hashemi, H.M.B. and Al-Amoudi, O.S.B. 2018. A review on the angle of repose of granular materials. *Powder Technology*, 330: 397-417.

Amos, R.T., Blowes, D.W., Bailey, B.L., Sego, D.C., Smith, L. and Ritchie, A.I.M. 2015. Waste-rock hydrogeology and geochemistry. *Applied Geochemistry*, 57: 140-156.

Andjelkovic, V, Pavlovic, N., Lazarevic, Z. and Radovanovc, S. 2018. Modelling of shear strength of rockfills used for the construction of rockfill dams. *Soils and Foundations.* 58(4): 881-893.

Anterrieu, O., Chouteau, M. and Aubertin, M. 2010. Geophysical characterization of the large-scale internal structure of a waste rock pile from a hard rock mine. *Bulletin of Engineering Geology and the Environment*, 69(4): 533-548.

Asaf, Z., Rubinstein, D. and Shmulevich, I. 2005. Determination of discrete element model parameters using in-situ tests and inverse solution techniques. Proceedings of the 15th international conference of the ISTVS, Hayama, Japan, 25–29.

Aubertin, M. 2013. Waste rock disposal to improve the geotechnical and geochemical stability of piles. *Proceedings of the world mining congress*, Montreal, Quebec, Canada.

Azam, S., Wilson, G.W., Herasymuik, G., Nichol, C. and Barbour, L.S. 2007. Hydrogeological behaviour of an unsaturated waste rock pile: a case study at the Golden Sunlight Mine, Montana, USA. *Bulletin of Engineering Geology and the Environment*, 66: 259–268.

Chung, Y.C. and Ooi, J.Y. 2008. A study of influence of gravity on bulk behaviour of particulate solid. *Particuology*, 6(6): 467-474.

Coetzee, C.J. 2016. Calibration of the discrete element method and the effect of particle shape. *Powder Technology*, 297: 50-70.

Cundall P.A. and Strack O.D.L. 1979. A discrete numerical model for granular assemblies. *Géotechnique*, 29(1): 47–65.

Derakhshani, S.M., Schott, D.L. and Lodewijks, G. 2013. Reducing computation time of dem simulations of fine granular materials, *Bulk Solids Handling*, 33: 52–56.

Derakhshani, S.M., Schott, D.L. and Lodewijks, G. 2015. Micro–macro properties of quartz sand: Experimental investigation and DEM simulation, *Powder Technology,* 269:127-138.

Fala, O., Molson, J., Aubertin, M., and Bussière, B. 2005. Numerical Modelling of Flow and Capillary Barrier Effects in Unsaturated Waste Rock Piles. *Mine Water and the Environment*, 24(4): 172-185.

Fala, O., Molson, J., Aubertin, M., Dawood, I., Bussière, B., and Chapuis, R.P. 2012. A numerical modelling approach to assess long-term unsaturated flow and geochemical transport in a waste rock pile. *International Journal of Mining, Reclamation and Environment, 27*(1): 38-55.

Franco, Y., Rubinstein, D. and Shmulevich, I. 2005. Determination of discrete element model parameters for soil-bulldozer blade interaction. Proceedings of the 15th international conference of the ISTVS, Hayama, Japan, 25–29.

Gamache-Rochette, A. 2004. Une étude de caractérisation en laboratoire et sur le terrain des écoulements de l'eau dans les roches stériles. Mémoire de maîtrise en sciences appliquées, École Polytechnique de Montréal, Département des génies civil géologique et des mines.

Gray, J. M. N. T. 2018. Particle segregation in dense granular flows. *Annual Review of Fluid Mechanics*, 50:407–433

Grima, A.P. and Wypych, P.W. 2011. Development and validation of calibration methods for discrete element modelling. *Granular Matter*, 13: 127-132.

Hajizadeh Namaghi, H., Li, S. and Jiang, L. 2015. Numerical simulation of water flow in a large waste rock pile, Haizhou coal mine, China. *Modeling Earth Systems and Environment*, 1(1-2).

Itasca Consulting Group, Inc. 2020. PFC — particle flow code in 2 and 3 dimensions. Version 6.0, Documentation set of version 6.00.15. Minneapolis, Minnesota, USA.

Jiang, Y., Wang, L., Wu, Q., Sun, Z., Elmo, D. and Zheng, L. 2020. An improved shear stress monitoring method in numerical direct shear tests by particle flow code, *Journal of Testing and Evaluation* (in press).

Jiang, Y.J., Fan X.Y., Li T.H. and Xiao, S.Y. 2018. Influence of particle-size segregation on the impact of dry granular flow. *Powder Technology,* 340: 39-51.

Just, S., Toschkoff, G., Funke, A., Djuric, D., Scharrer, G., Khinast, J. and Kleinebudde, P. 2013. Experimental analysis of tablet properties for discrete element modeling of an active coating process. *AAPS PharmSciTech,* 14(1): 402-411.

Kesseler, M., Heller, V. and Turnbull, B. 2018. A laboratory numerical approach for modelling scale effects in dry granular slides. *Landslides*, 15: 2145–2159.

Lahmira, B., Lefebvre, R., Aubertin, M. and Bussiere, B. 2016. Effect of heterogeneity and anisotropy related to the construction method on transfer processes in waste rock piles. *Journal of Contaminant Hydrology*, 184: 35-49.

Lahmira, B., Lefebvre, R., Aubertin, M. and Bussiere, B. 2017. Effect of material variability and compacted layers on transfer processes in heterogeneous waste rock piles. *Journal of Contaminant Hydrology*, 204: 66-78.

Marigo, M., Stitt, E.H. 2015. Discrete element method (DEM) for industrial application: comments on calibration and validation for the modelling of cylinder pellets. *Powder and Particle*, 32: 236-252.

Molson, J. W., Fala, O., Aubertin, M. and Bussiere, B. 2005. Numerical simulations of pyrite oxidation and acid mine drainage in unsaturated waste rock piles. *Journal of Contaminant Hydrology*, 78(4): 343-371.

Nichol, R. S. 1986. Rock segregation in waste dumps. Proceedings of the international conference on flow-through rock drains.

Plassiard, J.P., Belheine, N. and Donzé, F.V. 2009. A spherical discrete element model: calibration procedure and incremental response. *Granular Matter*, 11(5): 293-306.

Santos, D. A., Barrozo, M. A. S., Duarte, C. R., Weigler, F. and Mellmann, J. 2016. Investigation of particle dynamics in a rotary drum by means of experiments and numerical simulations using DEM. *Advanced Powder Technology*, 27(2): 692-703.

Su, H., Fu, Z., Gao, A. and Wen, Z. 2019. Numerical Simulation of Soil Levee Slope Instability Using Particle-Flow Code Method. *Natural Hazards Review,* 20(2): 04019001-14.

Sylvain, K., Pabst, T. and Dimech, A. 2019. Waste rock valorization in monolayer covers with elevated water table. *18th Global Joint Seminar on Geo-Environmental Engineering*, Montreal, Quebec, Canada.

Utili, S., Zhao, T. and Houlsby, G.T. 2015. 3D DEM investigation of granular column collapse: evaluation of debris motion and its destructive power, *Engineering Geology*, 186:3–16.

Valentino, R. Barla, G. Monstrasio, L. 2008. Experimental analysis and micromechanical modelling of dry granular flow and impacts in laboratory fume tests, *Rock Mechanics and rock engineering,* 41(1): 153–177.

Wensrich, C. M. and Katterfeld, A. 2012. Rolling friction as a technique for modelling particle shape in DEM. *Powder Technology*, 217: 409-417.

Yan, Z., Wilkinson, S.K., Stitt, E.H. and Marigo, M. 2015. Discrete element modelling (DEM) input parameters: understanding their impact on model predictions using statistical analysis, *Computational Particle Mechanics*, 2(3): 283–299.

Zhu, H.P., Zhou, Z.Y., Yang, R.Y. and Yu. A.B. 2008. Discrete particle simulation of particulate systems: A review of major applications and findings. *Chemical Engineering Science*,63(23): 5728-5770.