Seismic Assessment of Embankment Dams Using a Novel Energy-Based Approach

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ABSTRACT
In this study the step by step procedure is introduced to evaluate the dynamic behavior of an embankment dams under seismic loading, using a coupled energy-based pore pressure model. The simulated dam with 12 m height was assumed to be in Saguenay region as one of the most seismically active regions of eastern Canada. To define the dynamic behaviour of constituent materials of the simulated dam the energy-based model was combined with Sigmoidal-model in FLAC 2D software. The proposed models were developed, calibrated and verified based on a series of stress-controlled cyclic direct simple shear (DSS) tests performed on representative constituent materials of the dam at the Université de Sherbrooke. The results of all dynamic simulations using different implemented ground motions were compared and the dynamic behavior of the dam under seismic loadings was evaluated.

RÉSUMÉ
Dans cet article, une procédure étape par étape est introduite pour évaluer le comportement dynamique d'un barrage sous charge sismique, en utilisant du concept d'énergie. Le barrage simulé, d'une hauteur de 12 m, a été supposé se trouver dans la région du Saguenay, l'une des régions les plus actives sur le plan sismique de l'est du Canada. Pour définir le comportement dynamique des matériaux constitutifs du barrage, le concept d'énergie est combiné avec le modèle sigmoïdal dans le logiciel FLAC 2D. Les modèles proposés ont été développés, calibrés et vérifiés sur la base d'une série d'essais de cisaillement simple direct (DSS) cyclique à contrainte contrôlée été effectués à l'Université de Sherbrooke. Les résultats de toutes les simulations dynamiques utilisant différents mouvements du sol ont été comparés et le comportement dynamique du barrage sous les charges sismiques a été évalué.

1 INTROCUDTION

Although earthquakes occur in all regions of Canada, certain areas have a higher probability of experiencing damaging ground motions caused by earthquakes. In these areas, the rigorous assessment of the dynamic behavior of earth dams and their performance improvements are crucial to eliminate the risk of their failures under seismic loadings. A comprehensive understanding of the site condition, local seismic characteristics and dynamic behaviors of constituent materials of a simulated dam is of outmost importance to perform a reliable dynamic assessment.

The expert assessment of regional seismicity and the selection of appropriate site-specific earthquake design parameters are vital components of important projects located in seismic regions. The seismic hazard is described by spectral-acceleration values at different periods (e.g., 0.2, 0.5, 1.0 and 2.0 seconds). Ground motion probability values are also given in terms of probable exceedance, that is the likelihood of a given horizontal acceleration or velocity being exceeded during a particular period (e.g., 0.000404 per annum, equivalent to a 2-per-cent probability of exceedance over 50 years).

The compactness condition and consistency of materials at different locations of a dam can be identified using in-situ geophysical surveys (e.g., Spectral Analysis of Surface Wave, Modal Analysis of Surface Wave, and Multi Modal Analysis of Surface Wave methods) and geotechnical measurements (i.e., N-Standard Penetration Test, q_c-Cone Penetration Test). These surveys can thereafter be used to reconstitute the laboratory samples at the same in-situ density.

In recent decades, several total and effective stress constitutive models have been developed in geotechnical engineering practice to evaluate nonlinear dynamic response of earth dams under seismic loading. Numerous research works have been done to predict more accurately the earthquake-induced pore water pressure in different materials at various physical conditions. The developed pore pressure models can be classified into stress-, strain-, and energy-based models (Karray et al 2015, Kashila 2019). In this study, for each constituent material of a dam the energy-based model is combined with Sigmoidal-model in FLAC 2D software to introduce a novel coupled energy-based pore pressure model. The proposed models were calibrated and verified, in terms of shear stress-strain response and excess pore pressure, based on a series of stress-controlled cyclic direct simple shear (DSS) tests performed on representative constituent materials of the dam at the Université de Sherbrooke.

In this article, the step by step procedure of evaluating the dynamic behavior of a dam located in a seismically active region using the novel coupled energy-based pore pressure model is described. To simplify the numerical model in this study, the simulated earth dam is considered as homogeneous in terms of the compactness and density. In first step, dynamic behaviors of constituent materials of the simulated dam (clay core and dense sand-gravel fill) were determined using cyclic simple shear apparatus. Thereafter, the developed dynamic behaviors of tested materials were calibrated and implemented into the numerical model in FLAC 2D. The simulated dam was subjected to different seismic loads compatible with seismicity of the region for an annual probability of 1/2,475 per year. At the end, the results of all simulations with different implemented ground motions were compared and the dynamic behavior of the dam under seismic loadings is evaluated.

2 NUMERICAL MODEL DESCRIPTION

The geometrical configuration of the dam considered in this numerical study is presented in Figure 1. The simulated dam has 12 m height with upstream and downstream slopes of 2.0 H:1.0 V. the designed water level is 10.80 m at the upstream of the dam and drops to 1.76 m after reaching its downstream. The core of the dam is composed by clay and the shell by sand and gravel. The dam is founded on the rock.

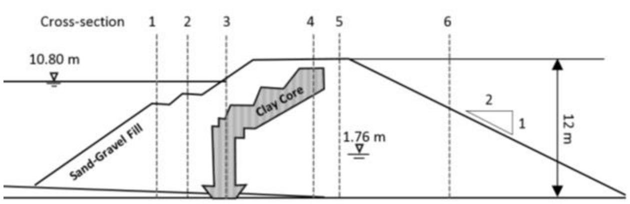


Figure 1. Geometrical configuration of the simulated dam

Dams and large reservoirs constructed on the area with high seismicity, pose a high-risk potential for downstream life and property. In this study, the simulated dam was assumed to be located in the Saguenay region which is in the most seismically active region of eastern Canada. The Saguenay region is located within ~100km distance from the seismically-active Charlevoix region, which has experienced repeated large earthquakes (M>6) in both the historical and geologic past (Tuttle and Atkinson, 2010). In addition to being near the Charlevoix zone (CHV), there is significant local seismicity, including the 1988 M5.8 Saguenay earthquake.

3 STRATEGY FOR SELECTING INPUT GROUND MOTIONS

In this section, the ground motions were selected compatible with Uniform Hazard Spectrum of the Saguenay region in the range of the natural periods of selected dam for an annual probability of 1/2,475 per year. According to the seismic hazard assessment of the region, an annual probability of 1/2,475 per year, selected ground motions have to represent events of magnitude 6.6 to 7.3 at distances of 27 km to 80 km.

In addition, due to the particular seismicity characteristics of the region, a number of spectral parameters were also used to select the most matched ground motions, recorded and synthetic, with that of the site under study. These parameters as well as their required ranges are mentioned below. These required ranges were determined according to the empirical correlations in literature (Chang and Krinitszky 1977, Trifunac and Brady, 1975, Cameron 2009, Lee, 2009, Lee and Green, 2010) and the seismic characteristics of recorded earthquakes in east of Canada.

- Predominant period, T_p: defines as the period corresponding to the maximum spectral acceleration of a motion and varies with the damping ratio (ξ). For this study, the T_p range was considered from 0.04 s to 1.12 s.
- Arias intensities, I_a: is a measure of the strength of a ground motion which relates the energy of the ground motion and the damage occurrence. In this study, the I_a range was considered between 0.7 m/s to 1.15 m/s.
- v_{max}/a_{max} ratio: was adopted by Tso et al. (1992) as a characteristic measure of the frequency content of a ground motion. This ratio is between 0.024 s to 0.043 s, in current study.
- Significant durations, $D_{5\text{-}75\%}$ and $D_{5\text{-}95\%}$: these parameters along with the amplitude and frequency content of a ground motions, significantly influences the response of geotechnical and structural systems. In this study, $D_{5\text{-}75\%}$ and $D_{5\text{-}95\%}$: can vary from 8 s to 15 s, and from 14 s to 25 s, respectively.

In addition to the seismic parameters presented above, eligible selected ground motions have to be matched and scaled with the target 2,475-Year spectral accelerations of the site, over the natural period range of the simulated dam. The natural period range of the dam was determined with the help of the software FLAC 2D using the procedure proposed by Karray and Lefebvre (2010). For this purpose, the stratigraphic profile of the dam and its foundation at different locations along the standard section was analyzed using a one-dimensional (1D) model. A half-sine pulse with a frequency of 50 Hz was applied to the base of each of these 1D models. The study of the model's numerical response to this impulse allowed us to estimate the natural frequency of the dam in different locations. Figure 1 shows 6 different cross sections selected to determine the natural period range of the simulated dam. Figure 2 presents the natural periods of 1-D soil deposits after applying a half-sine pulse to their bases. According to this figure, the determined natural period range of the dam is between 0.15 and 0.45 s.

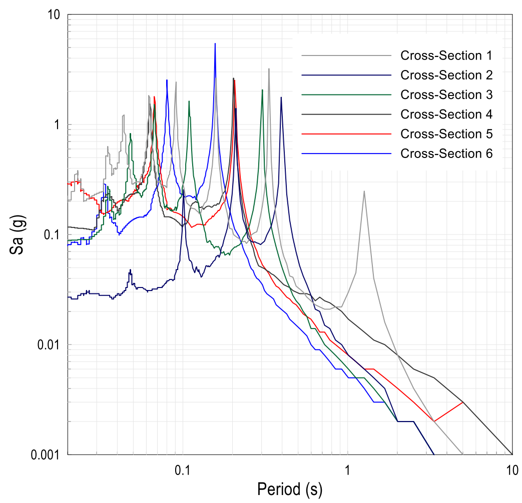


Figure 2. Natural period range determined by applying a half-sine pulse with a frequency of 50 Hz to the base of each 1D soil deposits

For the recurrence interval 0f 2,475 years, four (4) factored recorded and synthetic ground motions were selected compatible with the regional seismic conditions and spectral parameters. Recorded ground motions were selected from the PEER and NRCan databases, and the synthetics ones were chosen from the database of the simulated ground-motions for Eastern North America (ENA) published by Atkinson (2009).

The list of selected ground motions, with their corresponding spectral parameters and scaling factors are summarized in Table 1.

Table 1. List of recorded and synthetic ground motions compatible with the seismicity f the site

Parameter	Ground motions			
	Real		Synthetic	
	Iwate Japan	Nahanni Canada	East7a1 18	East7a2 3
Station	Miyagi great village	1	---	---
Component	EW (H2)	EN2	---	---
Scaling factor	0.67	0.38	0.88	2.05
Mw	6.90	6.76	7	7
Distance	41.13	9.60	20.60	41.6
I_a	0.80	3.9	4.31	5.10
v_{max}/a_{max}	0.058	0.029	0.049	0.042
$D_{5\text{-}75\%}$	9.90	7.2	5.31	7.71
$D_{5\text{-}95\%}$	17.70	7.5	7.29	9.17

Figure 3 presents the 2,475-year target response spectrum of the site as well as spectral accelerations of selected ground motions. As shown in this figure, the spectrum of selected ground motions agree well with the target S_a values at different periods, especially in the T_0 range of the dam. Moreover, the mean spectrum of the selected ground motions sufficiently reproduces the envelope of the site design spectrum.

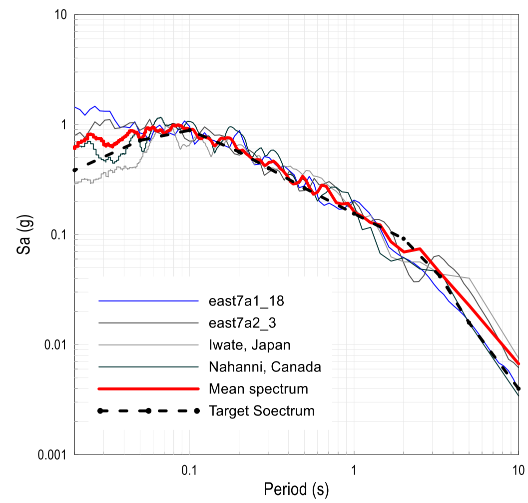


Figure 3. Target 2,475-year spectral accelerations and spectral acceleration curves of factored selected ground motions

4 NUMERICAL DYNAMIC ANALYSIS

In this study, the *fast Lagrangian analysis of continua* in two-dimensions numerical platform (FLAC 2D) was employed to investigate the seismic response of a dam by considering the nonlinear Sig 4 model and the effect of generated pore pressure during earthquake excitation. FLAC 2D is an explicit finite difference program contains built-in constitutive models and allows creating and implementing user-defined constitutive models using the FISH language to capture soil behavior in response to different load conditions and boundary restrains.

4.1 Nonlinear behavior of constituent materials of the dam (Sig 4 model) during cyclic loading

The experimental hysteresis stress-strain relationships determined from the cyclic DSS tests have been matched with the sigmoid function (Sig 4 soil model) available in the FLAC 2D library. The Sig 4 model is one of the soil models that have been proposed to predict the cyclic soil response over a wide range of strains. These models are also capable of simultaneously matching shear modulus and damping data, thus reducing the issue of damping overestimation at higher strains. In fact, the Sig 4 soil model is a 4-parameters model thus it provides more flexibility/control in fitting the stiffness and damping data compared to other models. The backbone curve of the Sig 4 soil model is given by equation [1]:

$$\tau = G_0 \left[y_0 + \frac{a}{1+exp(-(L_y - x_0)/b)} \right] \gamma \qquad [1]$$

where L_y is $Log_{10}(\gamma\%)$, and a, b, x_0, and y_0 are four curve-fitting parameters

For each material these parameters were selected so that to best match the experimental DSS data (Figure 4). The calibrated Sig 4 model parameters from the induced cyclic stress-controlled DSS hysteresis loops of both materials are listed in Table 2.

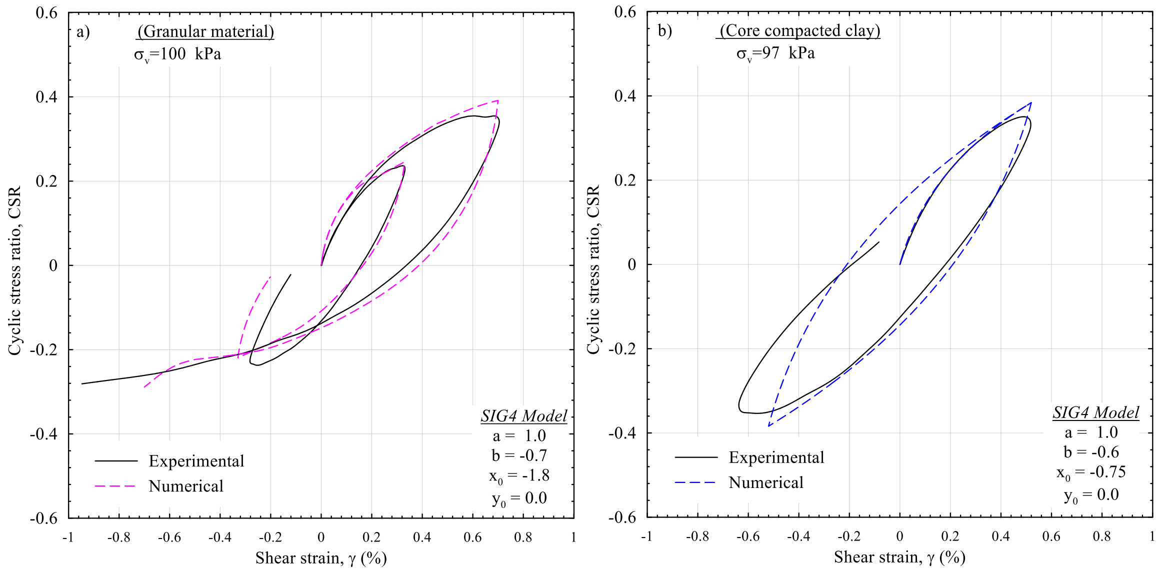


Figure 4. Comparisons between the experimental stress-strain hysteresis loops with those of the Sig 4 model for clay core and sand-gravel fill

Table 2. Sig 4 model parameters

Material	Parameters			
	a	b	x_0	y_0
Clay Core	1.0	-0.6	-0.75	0.0
Sand-Gravel	1.0	-0.7	-1.8	0.0

4.2 Energy-based pre water pressure model of constituent materials of the dam

A variety of models have been developed to predict the residual pore water pressure generation in saturated soils subjected to earthquake loading. Most of these models are currently used in computer codes to capture the dynamic response of the soil. Early developed models were based on two main approaches: cyclic stress (Seed et al. 1975) and cyclic strain (Martin et al. 1975) approaches. Another model termed strain energy-based model was developed by Nemat-Nasser and Shokooh (1979). Energy-based pore pressure models typically relate the ratio of excess pore pressure (R_u) generated during shearing to normalized unit energy, W_s that can be defined as the energy dissipated per unit volume of soil divided by the initial effective confining pressure (equation [2]). The dissipated energy per unit volume for a soil can be determined by integrating the area bound for each stress–strain hysteresis loops, as suggested by Green et al. (2000) and Yang and Pan 2018.

$$W_s = \frac{1}{2\sigma'_{mo}} \sum_{i=1}^{n-1} (\tau_{i+1} + \tau_i)\,(\gamma_{i+1} - \gamma_i) \qquad [2]$$

where W_s is the dissipated energy per unit volume of soil; τ_i is the applied shear stress at load increment i; γ_i is the shear strain at load increment i; σ'_{mo} is the initial mean effective stress; n is the total number of cycles.

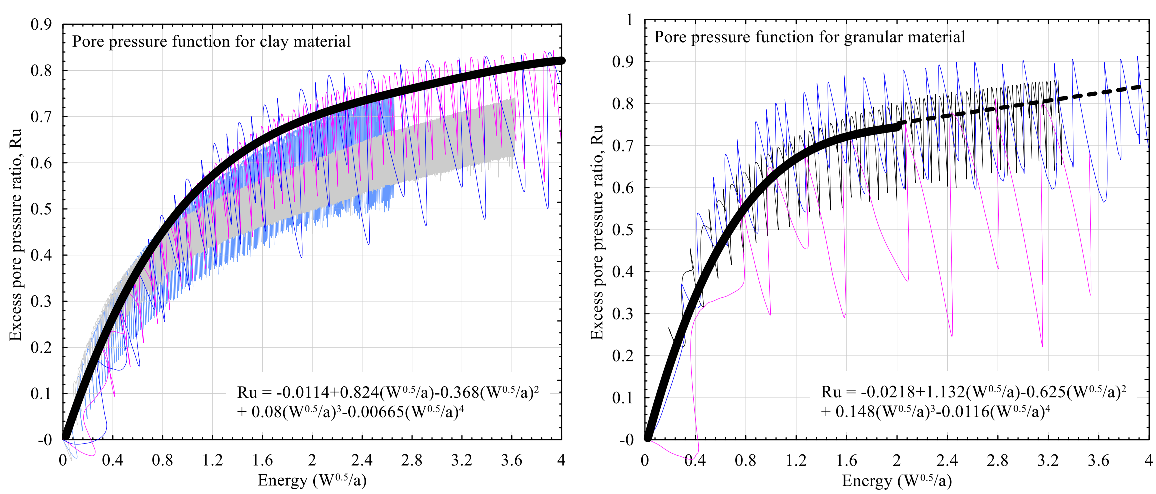


Figure 5. Correlations between the generated pore pressure and the normalized dissipated energy of (a) clay core (b) sand-gravel fill

The results of cyclic stress-controlled DSS tests were adapted to delineate a unique relation between pore pressure and the normalized dissipated energy $(W_s/\alpha)^{0.5}$ for each tested material. Figure 5 shows these relations for constituent materials of the dam which can be written in a general polynomial function as (equation [3]):

$$R_u = C_1\left(\frac{w_s^{0.5}}{\alpha}\right)+C_2\left(\frac{w_s^{0.5}}{\alpha}\right)^2 + C_3\left(\frac{w_s^{0.5}}{\alpha}\right)^3 \qquad [3]$$

where C_1, C_2, and C_3 and α are fitting and calibration parameters that determined from experimental cyclic DSS tests performed on materials (Figure 5).

The performance of developed Sig 4 models and the energy-based procedure represented above were examined against experimental data from the DSS tests on both materials shown in Figure 6. This figure shows a comparison between computed excess pore pressures and cyclic stress ratios with those measured in experiments. These comparisons indicate a good agreement between the measured and the computed soil response using aforementioned approach.

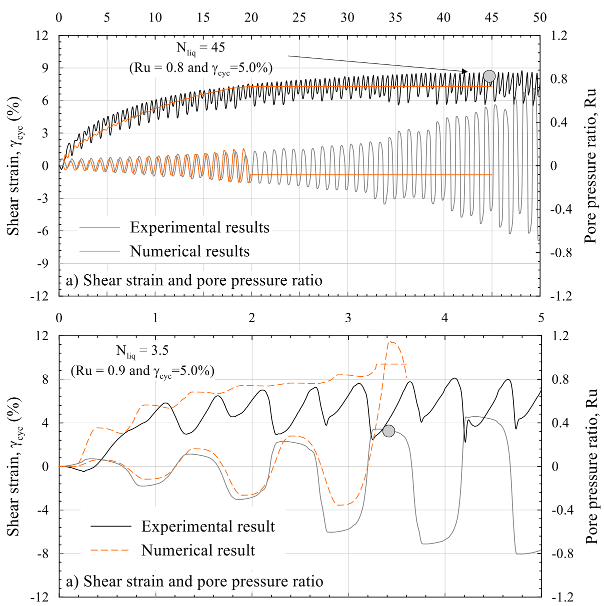


Figure 6. Comparison between computed excess pore pressures and cyclic stress ratios with those measured in experiments of (a) clay core (b) sand-gravel fill

In order to revalidate the accuracy of the proposed models, and the coherence of numerical lab scale results with experimental stress-controlled DSS tests, a comparison between their liquefaction potential (CRR-N_{liq}) were done in Figure 7. These results show a very good agreement between numerical and experimental DSS results of clay core and sand-gravel fill upon triggering the liquefaction (Ru= 0.9).

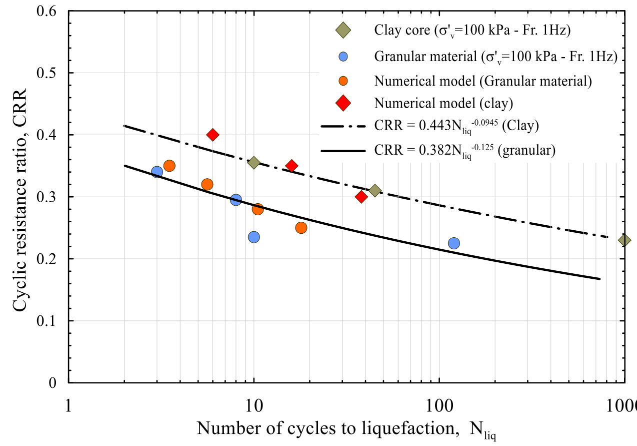


Figure 7. CRR-N_{liq} correlations of clay core and sand-grave fill using numerical and experimental results

5 DYNAMIC ANALYSIS RESULTS

At the first step, a mechanical and flow analysis were coupled to ensure the stability of the model in the general condition, before subjecting to seismic loading. Distributions of vertical effective stress, maximum shear modulus and pore pressure in the developed model after this state are shown in Figure 8.

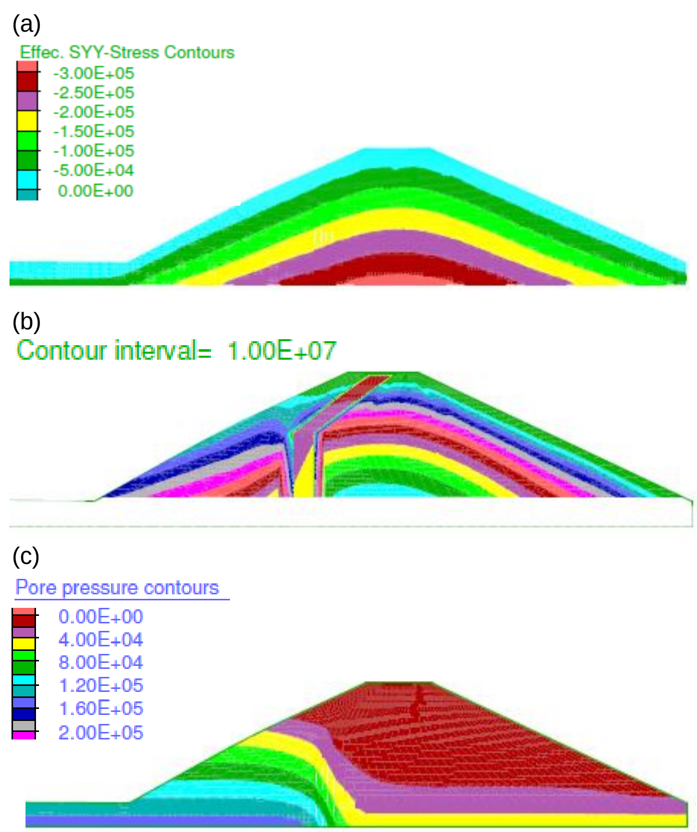


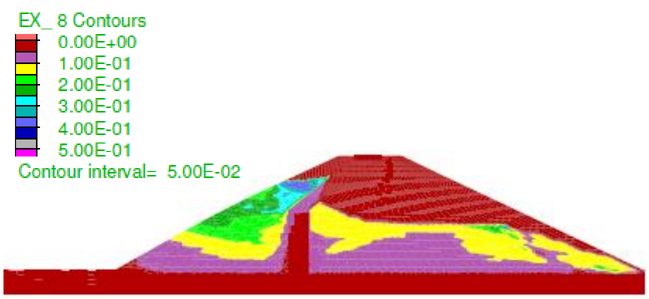
Figure 8. Distribution of (a) effective stress, (b) maximum shear modulus and (c) excess pore pressure after the static equilibrium state

After the static equilibrium state, dynamic analyses were performed by applying 4 different factorized accelerograms to the dam. The distribution of pore overpressure ratio, R_u, which is generally defined as the water overpressure generated by the earthquake divided by the initial effective vertical stress before loading, was recorded during the dynamic phase for each applying accelerogram. In addition, the distribution of maximum shear strain increments in each simulated model were used to determine probable surface instabilities. As an example, the distributions of pore overpressure ratio and maximum shear strain increments of the numerical model after applying ground motion of Iwate, Japan for the return periods of 1/2,475 years are shown in Figure 9.

The evolution of excess pore pressure in figure 9.a shows an increase of Ru value up to 0.4 at some region of the dam's upstream. The distribution of the pore pressure under the water level in downstream of them is less than 0.2.

According to figure 9.b, the shallow sliding surface can be observed at downstream slopes of the dam in granular materials with shear strains increments higher than 5%.

(a)



(b)

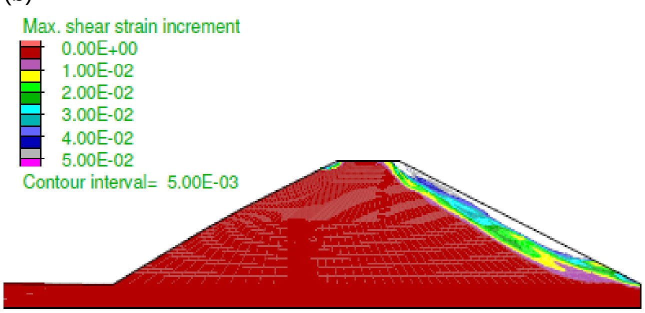


Figure 9. (a) Excessive pore pressure ratio and (b) Maximum shear strain increment of simulate dam subjected to the ground motion Iwate, Japan compatible the return period of 1/2,475 years

In addition to figures represent the distribution of deformation and pressures in the dam after the dynamic state, 5 additional control points also considered at different locations of the dam to observe the displacements and excessive pore pressures during the dynamic state (Figure 10). Two (2) control points were considered on the crest and downstream slope of the dam to evaluate the deformations. The excess pore pressure development was also monitored at one (1) point on the clay core and two (2) points on upstream and downstream fill materials. For these control points, the developed pore pressure ratio and maximum vertical and horizontal displacements at the end of the dynamic state are presented in Table 3.

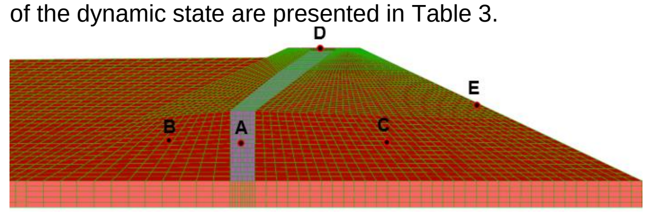


Figure 10. Location of control points in the numerical model

At the end of the seismic loading, the dam's crest had the slight horizontal displacement of around 5 cm (points D). the recorded horizontal displacement at point E located at the middle of down stream slope is more around 25 cm after the dynamic state. The vertical displacements at crests and middle of the downstream slopes are around 0.55 cm and 2 cm, respectively.

Table 3. Developed excess pore pressure and displacement at control points (1/2,475 years)

Location	Control point	Ground motions			
Excess pore pressure (Ru)					
		1	2	3	4
Clay-Core	A	0.04	0.10	0.07	0.08
Granular Fill – U*	B	0.17	0.22	0.19	0.24
Granular Fill – D**	C	0.10	0.12	0.14	0.10
Horizontal Displacement (x,y), cm					
Crest	D	5.00	5.20	5.10	4.90
Slope- D	E	25.00	24.95	25.25	25.00
Vertical Displacement (x,y), cm					
Crest	D	0.54	0.58	0.61	0.50
Slope- D	E	2.15	2.34	2.38	2.40

*D: Downstream **U: Upstream
1: Iwate, Japan 2: Nahanni, Canada 3: East7a1_18 4: East7a2_3

To have a further insight into the capability of the energy-based approach to capture the high excessive pore pressure and instabilities during and after the seismic loading, an additional simulation for the return period of the 1/10,000 years were performed.

For this purpose, the recorded ground motion of Iwate, Japan was selected and factored to be in general agreement with target spectral accelerations of 1/10,000 return period.

As can be seen in Figure11, by decreasing the recurrence interval to 1/10,000 years (with the higher mean-hazard ground motions and the peak ground acceleration PGA) some areas at the upstream of the dam reached the high R_u value of around 0.8. The distribution of maximum shear strain increments in dam for this return period is also presented in Figure 11. As can be seen in this figure, an explicit sliding surface can be observed at the downstream slope of the dam by increasing the reoccurrence interval, so that, the horizontal and vertical displacement at dykes' crest and at the middle of their downstream slopes increased significantly.

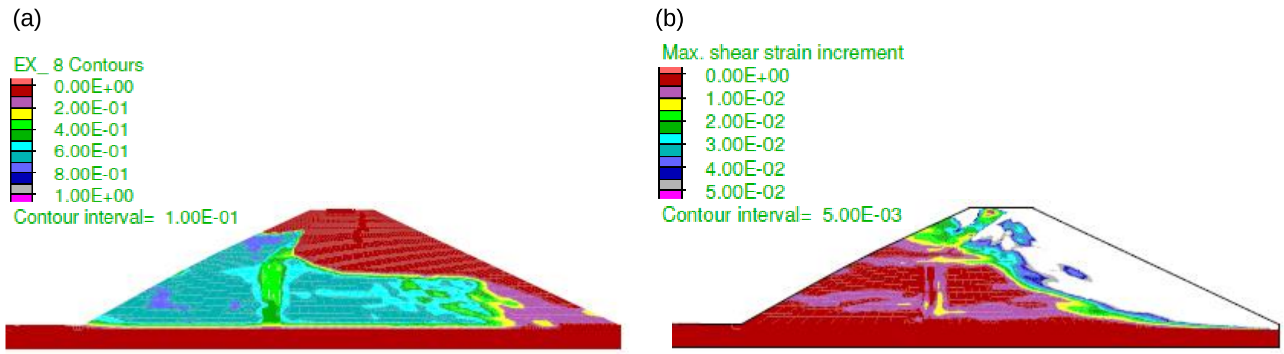


Figure 11. (a) Excessive pore pressure ratio and (b) Maximum shear strain increment of simulate dam subjected to the ground motion of Iwate, Japan compatible the return period of 1/10,000 years

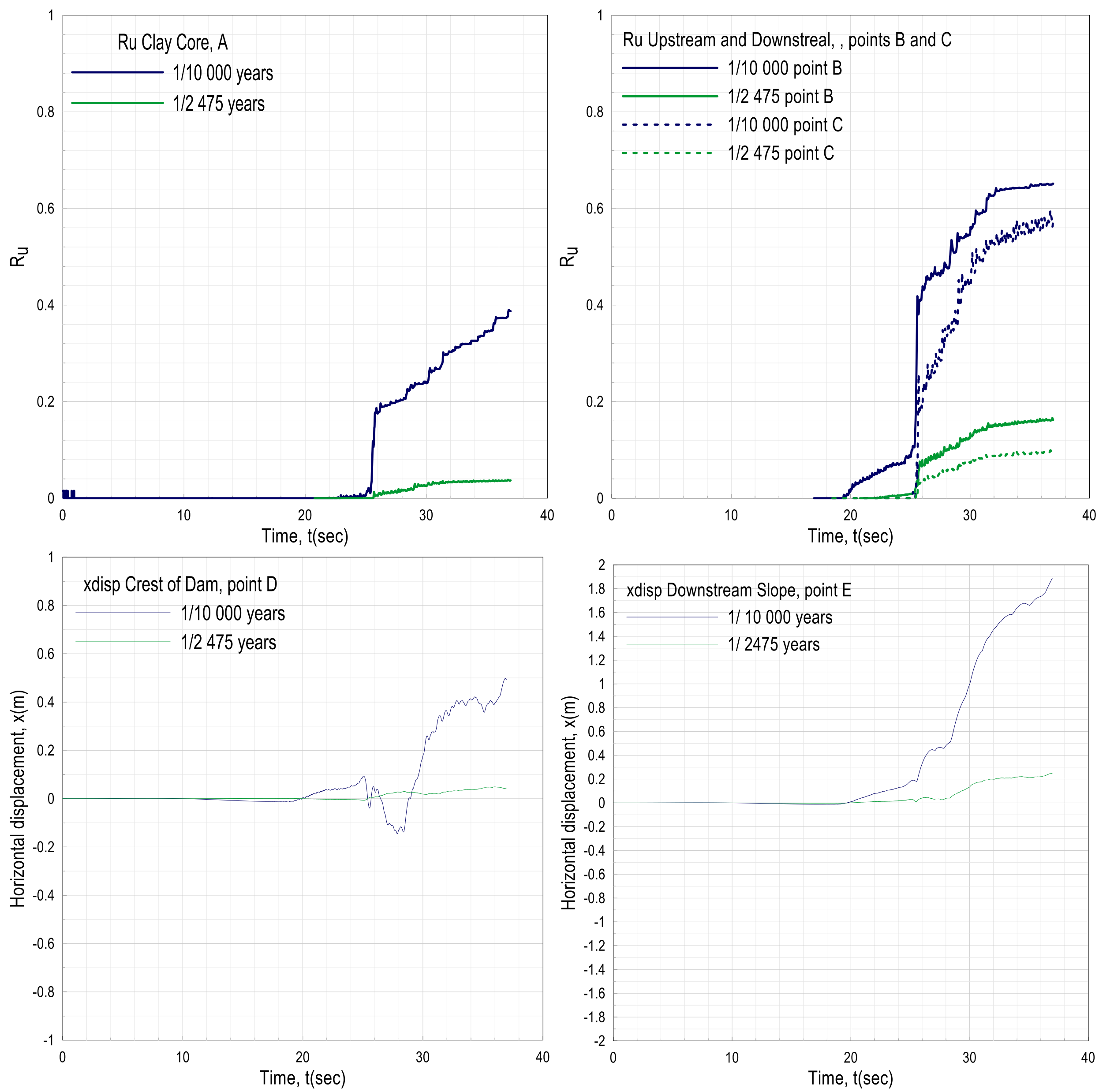


Figure 12. development of the excessive pore pressure inside (a) the clay core, (b) the granular fill - and horizontal displacement at (c) the crest and (d) the downstream slope of the simulated dam for different return periods of 1/2,475 years and 1/10,000 years (recorded ground motion: Iwate, Japan)

In Figure 12, development of excess pore pressure and displacement at control points during the proposed seismic loading are presented. In this figure, the results are compared with those of the return periods of 1/2,475. The excess pore pressure at the bottom of the clay core (point A), upstream (point B) and downstream (point C) of the dam considerably increased for return periods of 1/10,000 years.

6 SUMMARY AND CONCLUSION

In this study, the step by step procedure of evaluating the dynamic behavior of a dam located in a seismically active region using the novel coupled energy-based pore pressure model is described. The proposed models were developed and calibrated using a series of stress-controlled cyclic direct simple shear (DSS) tests performed on representative constituent materials of the dam at the Université de Sherbrooke. 4 different ground motions had been applied to the FLAC 2D model to assess the dynamic behavior of the simulated dam while subjecting to the seismic loads and for an annual probability of 1/2,475 per year. In order to evaluate this novel approach to simulate considerable excessive pore pressure development and instabilities in the dynamic model, an additional analysis was also performed for the return period of 1/10,000 years with the higher mean-hazard ground motions and the peak ground acceleration PGA. The distributions of excess pore pressure, maximum shear strain increment and displacement were evaluated for each simulation and the conclusion below had been achieved:

The distribution of maximum shear strain in the numerical model shows a shallow sliding surfaces at downstream of the dam, which resulted in 5 cm horizontal displacement of the crests of the dam and 25 cm of its downstream slope.

The mean-hazard ground motions for 5% damped horizontal and the peak ground acceleration PGA of the site for a return period of 1/10,000 years are almost twice as high as that those for return period of 1 / 2,475 years; these high parameters explains the high excessive pore pressure values and explicit sliding surface of the numerical model associated with the 1/10,000 return period.

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