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## Earth pressure coefficient in a vertical backfilled opening

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### ABSTRACT

A number of analytical solutions have been developed to estimate the stresses in backfilled openings such as retaining walls, silos, trenches and mine stopes. These solutions contain an earth pressure coefficient  $K$ , defined as the ratio of the horizontal to vertical effective (principal) stresses. For the case of mine backfilled stopes with immobile rock walls, some researchers proposed to use the Jaky's at-rest earth pressure coefficient  $K_0$  while others proposed to use the Rankine's active earth pressure coefficient  $K_a$ . To clarify this debate issue, the horizontal and vertical stresses at the base of a column were measured for different thicknesses of backfill. The values of  $K$  were obtained through the measured stresses. The results show that the value of  $K$  is close to the Rankine's active earth pressure coefficient  $K_a$  at the center and the backfill can approach an active state during the placement with even immobile confining walls.

### RÉSUMÉ

De nombreuses solutions analytiques ont été développées pour estimer les contraintes dans les ouvertures remblayées telles que les murs de soutènement, les silos, les tranchées et les chantiers miniers. Ces solutions contiennent un coefficient de pression des terres  $K$ , défini comme le rapport entre les contraintes effectives (principales) horizontales et verticales. Pour le cas de chantiers remblayés des mines avec des murs rocheux immobiles, certains chercheurs ont proposé d'utiliser le coefficient de pression des terres au repos de Jaky  $K_0$  tandis que d'autres ont proposé d'utiliser le coefficient de pression des terres actif (ou poussé) de Rankine  $K_a$ . Pour clarifier ce point de débat, les contraintes horizontales et verticales à la base d'une colonne ont été mesurées pour différentes épaisseurs de remblai. Les valeurs de  $K$  ont été obtenues à partir des contraintes mesurées. Les résultats montrent que la valeur de  $K$  au centre de la colonne est proche du coefficient de pression des terres actif de Rankine  $K_a$  et le remblai peut approcher un état actif pendant le placement avec des murs de confinement même immobiles.

### 1. INTRODUCTION

Estimation of the pressures and stresses in backfilled openings is a critical concern in the design of the confining structure such as retaining walls, silos, trenches and mining backfilled stopes. When a backfill is placed in a confining structure, the backfill will settle down under its own weight. The rigid surrounding walls tend to hold the backfill in the original place, leading to generation of shear stress along the contact areas between the backfill and surrounding walls. Part of the load of the backfill is transferred to the surrounding walls, resulting in lower backfill pressures than those calculated based on the overburden solution. This phenomenon is known as arching effect (Janssen

1895). By considering arching effect, Marston (1930) proposed the following equations to evaluate the vertical ( $\sigma_{vH}$ ) and horizontal ( $\sigma_{hH}$ ) stresses applied on top of the conduits buried in trenches:

$$\sigma_{vH} = \frac{\gamma B}{2K \tan \delta} \left[ 1 - \exp\left(-\frac{2K \tan \delta}{B} H\right) \right] \quad [1]$$

$$\sigma_{hH} = \frac{\gamma B}{2 \tan \delta} \left[ 1 - \exp\left(-\frac{2K \tan \delta}{B} H\right) \right] \quad [2]$$

where  $\gamma$  is the unit weight of the backfill;  $B$  is the width of the slope;  $\delta$  is the friction angle along the interfaces between the backfill and surrounding rock walls;  $H$  is the thickness of the backfill placed on top of the conduits;  $K$  is the earth pressure coefficient, which is defined as the ratio of the horizontal ( $\sigma_h$ ) to vertical ( $\sigma_v$ ) principal stresses ( $K = \sigma_h/\sigma_v$ ).

The presence of arching effect has been reported in many fields of geotechnical engineering. Examples include the placement of backfill behind retaining walls (Take and Valsangkar 2001; Paik and Salgado 2003), storage of granular materials in silos (Blight 1986; Ooi and Rotter 1990), pour of slurry in trenches (Ruffing et al. 2010; Li et al. 2015), slope backfilling in underground mines (Askew et al. 1978; Knutsson 1980; Aubertin et al. 2003; Li et al. 2003, 2005; Pirapakaran and Sivakugan 2007b; Thompson et al. 2012).

Over the years, a number of analytical solutions have been proposed to estimate the stresses in mine backfilled slope by taking into account more realistic conditions, such as the pore water pressure (Li and Aubertin 2009a, 2009b; Zheng et al. 2019, 2020), three dimensional geometry (Li et al. 2005; Pirapakaran and Sivakugan 2007a), non-linear stress distribution across the width of the slope (Li and Aubertin 2008, 2010), and inclination of slopes (Caceres 2005; Ting et al. 2011, 2012; Jahanbakhshzadeh et al. 2017, 2018a, 2018b). All these analytical solutions need the knowledge of earth pressure coefficient  $K$ , which can significantly influence the calculated vertical and horizontal stresses.

For the case of mine backfilled stopes with immobile rock walls, some researchers (Pirapakaran and Sivakugan 2007a, 2007b; Fahey et al. 2009; Ting et al. 2011) proposed to use the Jaky's at-rest earth pressure coefficient  $K_0$  while others proposed to use the Rankine's active earth pressure coefficient  $K_a$  according to the numerical simulations and some comparisons with experimental results (e.g., Li et al. 2003, 2005; Li and Aubertin 2009a, 2009b; Ting et al. 2012) even though the confining walls are immobilized.

To recall that the concept of the earth pressure coefficient was initially proposed in soil mechanics to evaluate the horizontal stress on retaining wall constructed to retain natural soil (Coulomb 1776; Rankine 1857; Jaky 1944; Terzaghi et al. 1996). The retained natural soil is initially in an at-rest state before any disturbance associated with an excavation and the construction of retaining wall. Due to this initial state condition assumed on the in-situ soil, an at-rest state yields as long as the retaining wall is kept immobile. The Jaky's earth pressure coefficient  $K_0$  can then be used (Jaky 1948):

$$K_0 = 1 - \sin \phi \quad [3]$$

where  $\phi$  ( $^\circ$ ) is the internal friction angle of the soil.

When the retaining wall is allowed to move away from the retained soil, the horizontal stress  $\sigma_h$  diminishes. If the movement of the retaining wall is large enough, the horizontal stress can reduce to such

degree that the Mohr circle of the stress state meets the Coulomb yield envelop. The retained in-situ soil yields and reaches an active state. The Rankine's active earth pressure coefficient  $K_a$  applies as follows (CGS 2006; McCarthy 2007; Das 2010):

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) \quad [4]$$

For the cases of backfilled openings, the problem is completely different from that of a retaining wall, built to hold an in-situ soil after an excavation. In mine stopes, the granular material to be retained or confined is a backfill, not a natural in-situ soil. The confining structures pre-exist before the placement of any backfill. The initial state of the backfill after its placement in the confining structures remains unknown. The immobilization of the confining walls cannot be considered as a necessary and sufficient condition for the backfill placed in the confining structure to be in an at-rest state.

Recently, Sobhi et al. (2014, 2017) showed once again by numerical modeling that the horizontal to vertical (principal) stress ratio along the center line of vertical backfilled stopes is close to the Rankine's active earth pressure coefficient. They further showed that the backfill placed in the stope can reach yield state even though the rock walls remain immobile after the placement of the backfill. Yang et al. (2017) extended the numerical simulations to investigate the values of  $K$  at the stope center and near the wall. They showed that the value of  $K$  near the opening center is close to  $K_a$  when the backfill internal friction angle ( $\phi$ ) or Poisson's ratio ( $\mu$ ) is smaller than their respective critical value. Otherwise, the value of  $K$  is close to the at-rest earth pressure coefficient defined by the Poisson's ratio ( $K_0$ ) $_\mu$  [ $= \mu/(1-\mu)$ ], rather than the Jaky's at-rest earth pressure coefficient (Eq. 3). Near the wall, the value of principal stress ratio  $K_{ps}$  (ratio between the minor and major principle stresses) is close to  $K_a$  while the value of  $K$  (ratio between the horizontal and vertical stresses) at the bottom of the stope is close to ( $K_0$ ) $_\mu$ . The mechanism for why the backfill placed in a confining structure can be in an active or at-rest state was given in Yang et al. (2018), who further indicated that it is necessary to measure the Poisson's ratio in order to determine the state of the backfill placed in confining structures.

In order to verify the validity of the different theories by experimental results, laboratory tests were conducted with simultaneous measurement of the vertical and horizontal stresses at the base level of a vertical cylinder column with different thicknesses of backfill. The values of  $K$  were calculated with the measured horizontal and vertical stresses.

## 2. LABORATORY TESTS AND RESULT INTERPRETATION

The tested material is a sand characterized by a specific gravity of  $G_s = 2.82$ , a coefficient of uniformity  $C_u = 2.6$  and a coefficient of curvature of  $C_c = 1.1$ . At the loosest state, it has a density of  $\rho = 1680 \text{ kg/m}^3$  and an internal friction angle of  $\phi = 35^\circ$ .

Figure 1 shows a schematic presentation of the testing instrumentation. It is composed of a Plexiglas column and two miniature stress sensors. The Plexiglas column has a height of 50 cm and a diameter of 15.5 cm. The stress sensors have a diameter of 6.5 mm and thickness of 1 mm. The two stress sensors were placed at the center of the base level of the column. They were calibrated by filling water in the column before and after each backfilling test.

In order to minimize the impact of sand placement, the filling operation was made very carefully. The falling height was kept almost constant in order to obtain a uniform backfill in the column. Stress readings were made after each addition of backfill 10 cm thick.

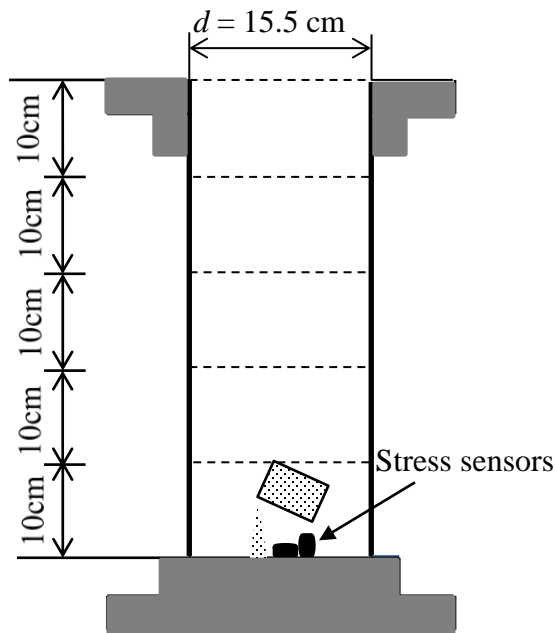


Figure 1. Schematic diagram of the testing instrumentation (not in scale)

Figure 2 shows the variation of the vertical and horizontal stresses as function of the thickness of the filled sand measured at the center of the column. Those calculated by applying the analytical solution (Eqs. 1 and 2) using  $K_a$  and  $K_0$  are also shown in the figure. It can be seen that the measured vertical stresses at the center are smaller than the overburden pressure, indicating the occurrence of arching effect. The measured horizontal stresses at the center agree well with those calculated by the analytical solution using either  $K_0$  or  $K_a$  (somewhat closer with  $K_0$ ). While the measured vertical stresses at the center agree well

with those calculated by the arching solution using  $K_a$ . These results are consistent with the numerical results of Li et al. (2003) and Sobhi et al. (2016).

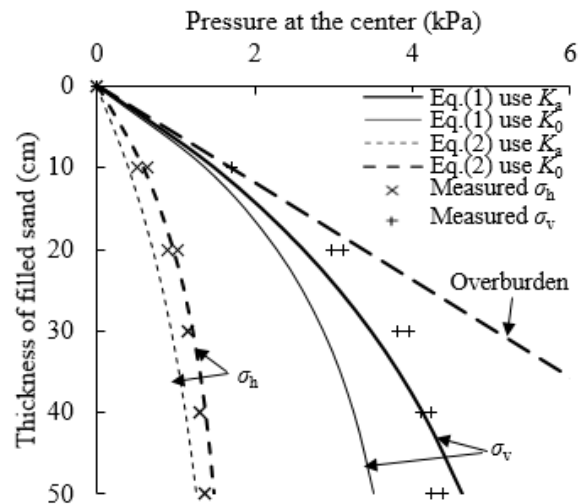


Figure 2. Variation of the vertical and horizontal stresses as a function of the fill thickness at the center of the column, measured and calculated with the arching analytical solution [Eqs. (1) and (2)] using Rankine's active ( $K_a$ ) and Jaky's at-rest ( $K_0$ ) earth pressure coefficients; on the figure are also plotted the vertical stresses based on the overburden solution.

Figure 3 shows the earth pressure coefficient  $K$  at the center of the column, calculated using the measured horizontal and vertical stresses. The Jaky's at-rest earth pressure coefficient  $K_0 (= 1 - \sin 35^\circ = 0.43)$  and Rankine's active earth pressure coefficient  $K_a (= (1 - \sin 35^\circ)/(1 + \sin 35^\circ) = 0.27)$  are also included on the figure.

At the column center, the results show that the value of  $K$  changes with the thickness of the backfill. At the beginning of the filling operation, it takes a value between the Rankine's active earth pressure coefficient  $K_a$  and Jaky's at-rest earth pressure coefficient  $K_0$ . When the fill thickness increases, the value of  $K$  decreases and tends to stabilize at a value close to the Rankine's active earth pressure coefficient  $K_a$ . These results are consistent with the previous numerical results of Li et al. (2003) and Sobhi et al. (2017). An active state of the backfill along the center line of the opening is possible during the backfilling even with immobile confining walls.

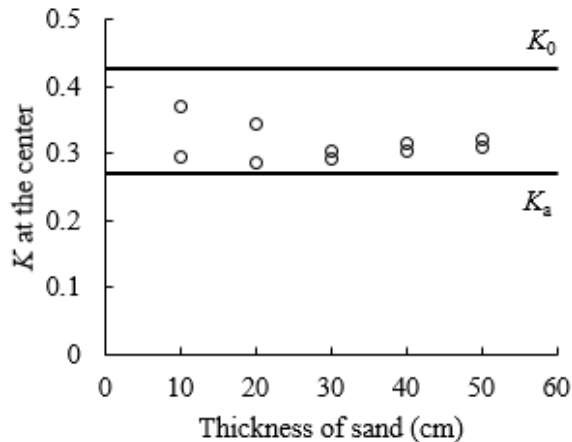


Figure 3. The earth pressure coefficients  $K$  calculated from the measured horizontal and vertical stresses at the center; the Rankine's active ( $K_a$ ) and Jaky's at-rest ( $K_0$ ) earth pressure coefficients plotted on the figure.

### 3. CONCLUSION AND DISCUSSION

A series of laboratory tests were conducted to measure the vertical and horizontal stresses at the base level at the center during the filling of a column with a sand. The earth pressure coefficient  $K$  were calculated from the measured horizontal and vertical stresses. The results show that the value of  $K$  is close to the Rankine's active earth pressure coefficient ( $K_a$ ) at the center of the column. The previous numerical results of Li et al. (2003) and Sobhi et al. (2017) have thus been confirmed by present experimental results. The experimental results show that an active state is totally possible in a backfilled opening even though the confining wall remains immobilized during the filling of the confining structure. However, it should be noted the test results were obtained with a dry backfill placed in a vertical column without vibration. More experiment work can be necessary to take into account pore water pressure, stope wall inclination, and mine production blasting vibration.

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