

EFFECT OF HEIGHT AND DOWNSTREAM SLOPE ON THE SEISMIC BEHAVIOR OF TAILINGS IMPOUNDMENTS REINFORCED WITH WASTE ROCK INCLUSIONS

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ABSTRACT

Many tailings storage facilities are designed with retaining dikes built using the upstream-raised construction method, commonly used due to its simplicity and cost-effectiveness. However, the stability of such type of impoundments remains a concern, as these have been prone to failure, especially in high seismicity regions. The use of waste rock inclusions (WRI) was proposed to improve the stability of tailings impoundments, particularly for those with upstream dikes. This article presents numerical analyses of the seismic behavior of tailings impoundments, considering cases reinforced with waste rock inclusions. The effects of the downstream slope and height of the impoundment on the seismic stability are investigated through a parametric study. The effect of each parameter is assessed individually using various indicators such as the critically displaced volume of tailings, and the deformation of the impoundment crest of the downstream slope. The simulations results are used to develop relationships to estimate the displacement of the slope as a function of waste rock inclusions configuration and geometry of the tailings impoundment.

RÉSUMÉ

De nombreuses digues de retenue de résidus miniers sont construites en utilisant la méthode de construction amont, souvent utilisée en raison de sa simplicité et de son faible coût. Cependant, la stabilité de ce type de digue demeure une préoccupation en raison des risques de défaillance, en particulier dans les régions à forte sismicité. L'utilisation d'inclusions de roches stériles (WRI) dans les parcs à résidus permet d'améliorer la stabilité des ouvrages, particulièrement ceux construits avec la méthode amont. Cet article présente des résultats d'analyses numériques du comportement sismique des digues de retenue des résidus, en considérant dans certains cas la présence d'inclusions. Les effets de la pente aval et de la hauteur de la digue sur la stabilité sismique sont étudiés via une étude paramétrique. L'effet de chaque paramètre est évalué individuellement à l'aide de divers indicateurs tels que le volume des résidus déplacés de façon critique et la déformation de la crête et de la pente des digues. Les résultats des simulations sont utilisés pour développer des relations afin d'estimer le déplacement de la pente en aval en fonction de la configuration des inclusions et de la géométrie du parc à résidus.

1 INTRODUCTION

Hard-rock mine tailings constitute one of the by-products of mining operation, and they are commonly deposited of in surface impoundments as a slurry. Such impoundments are often constructed using retaining dikes made, at least in part, of granular materials such as coarse-grained waste rock. During an earthquake, tailings may undergo strength loss due to generation of excess pore water pressure, a process known as liquefaction, which may lead to the failure of the impoundment due to additional loading on the retaining dikes. Even under static conditions, these structures are sometimes prone to failures, which may result in operation disruption, significant environmental damages and even loss of human lives (e.g. Aubertin et al. 2002; Roche et al. 2017).

The waste rock inclusion (WRI) method of co-disposal was proposed as a practical means to improve the geotechnical behavior of tailings impoundments (Aubertin et al., 2002). This method mainly consists of constructing continuous rows of waste rock, in parallel with tailings deposition and each dike raising, along selected routes to divide the impoundment into interconnected cells (James et al., 2013).

The effects of waste rock inclusions on the consolidation of tailings and stability of tailings impoundments have been studied in recent years using conventional and specialized laboratory testing, physical models and numerical simulations. The results of these studies indicate that the presence of WRI may improve significantly the geotechnical response of tailings impoundments under both static and dynamic loadings (James, 2009; L-Bolduc and Aubertin, 2014; Ferdosi et al., 2015; Saleh Mbemba, 2016).

The ongoing study presented here, which is part of a larger research program, aims at better quantifying the impact of waste rock inclusions in tailings impoundments and develop a method to optimize their configuration under seismic loading using numerical modeling. A brief description of the numerical models is provided before discussing key results. The article also presents the variation trends for the critically displaced volume of tailings, the deformation of the impoundment crest of upstream dikes, and the permanent displacement of the external slope for various WRI configurations. These results are used to develop a design approach for the optimum configuration of waste rock inclusions in tailings impoundments to improve their seismic stability.

2 DYNAMIC NUMERICAL ANALYSIS

The seismic behavior of tailings impoundments is analyzed using dynamic numerical analyses and evaluated based on excess porewater pressure, cyclic stress ratio, and the displacements pattern and magnitude. The finite-volume software FLAC (Fast Lagrangian Analysis of Continua) Version 8.00 (Itasca, 2016) is used in this study.

2.1 Analysis steps

A robust modeling procedure was established as the first step of the numerical study to simulate the seismic response of tailings impoundments with dikes built using the upstream method. The procedure consists of two stages. First, the static conditions are simulated to define the initial state of stresses, strains and pore water pressures, under mechanical and hydraulic equilibrium. The stress distribution from the static phase is then used as initial condition for the dynamic analysis.

2.2 Model geometry and boundary conditions

The simulations aim to analyse hard-rock tailings impoundments, with different heights and slopes, and WRI configurations, to assess the influence of each parameter on the seismic response. [Figure 1](#page-1-0) presents the general model geometry and distribution of material zones of a representative tailings impoundment with WRI considered for the parametric study. The starter dike and subsequent raises, each 2 m-high, are composed of waste rock. The crest width and height of the starter dike are 19 m and 10 m, respectively. The impoundment is modelled up to 140 m away from the dike crest. The potential of failure of such impoundments tends to occur near and along the downstream slope, so limiting the horizontal size of the model (beyond a minimum) doesn't affect the analysis,

while reducing the number of elements and the computational time. A 5-m-thick layer of moderately fractured bedrock is added beneath the impoundment.

The element size is defined based on the study from Kuhlemeyer and Lysmer (1973), in which they suggested to use elements with a maximum size of a tenth of the wavelength of the highest frequency component of the input wave that contains appreciable energy. This guideline is applied with the typical element size of 1 m by 1 m, which ensures that ground motions frequencies up to 20 Hz are realistically transmitted through the model.

The mechanical and fluid boundary conditions applied to the models during the static phase of the analysis consist of a fixed horizontal position on both sides of the model, no vertical displacement at the base and a fixed (nil) pore water pressure at the surface and on the right side of the model (i.e., external boundary of the dikes). Modeling a phreatic surface at the top of the impoundment represents a worst case scenario for dynamic stability, and it is also realistic as the water level within active tailings impoundments is often at, or above, the surface of the tailings. This condition is consistent with the high initial water content of the deposited slurry, relatively low hydraulic conductivity of tailings, and limited rainfall infiltration into the impoundments.

The boundary conditions considered for the dynamic analyses consist of a free field boundary on the sides of the models (i.e. in place of the fixed horizontal boundaries used in the static analysis) and the application of ground motion at the base of the model in the form of horizontal accelerations. Application of a free field boundary condition with the PM4Sand model used here has not been verified yet (Boulanger & Ziotopoulou, 2017). The free field boundary condition on the left side of the model is applied through a boundary condition element added as an outer column in the tailings [\(Figure 1\)](#page-1-0). The elastoplastic Mohr-Coulomb model is assigned to this element in both static and dynamic analyses. The fluid boundary conditions of the static phase were retained during the dynamic analyses.

- (1) Bedrock (2) Tailings (3) Waste rock inclusions
- (4) Upstream dikes (5) Starter dike
- (6) Boundary condition element

Figure 1. Conceptual model of a tailings impoundment with 3 waste rock inclusions.

2.3 Constitutive models and material properties

The elastoplastic Mohr-Coulomb model is used in the static stage to simulate the behavior of all the materials in the impoundment. The hydro-geotechnical properties of tailings such as unit weight, porosity and hydraulic conductivity are influenced by the method of placement, age and height of the deposit inside the impoundment. Various authors have evaluated the properties of hard rock mine tailings in the Abitibi region (Québec) from field or laboratory tests (e.g. Bussière 2007; L-Bolduc, 2012; Contreras Romero, 2013; Essayad, 2015; Grimard, 2018; Boudrias, 2018). The properties of the tailings considered in the numerical analyses, summarized in [Table 1,](#page-3-0) are based on these investigations.

The dry unit weight of the waste rock material (20 kN/m³) is selected based on the typical values reported in Peregoedova (2012), Peregoedova et al. (2013), Essayad et al. (2018), and Martin and Aubertin (2019). The value of the internal friction angle of the waste rock is based on values reported by Aubertin et al. (2013) and Maknoon (2016). The shear and bulk modulus of waste rock material are derived from the following relationships proposed by Seed et al. (1984) and Rollins et al. (1998):

$$
G = 55000. (0.6. \sigma'_{v})^{0.5}
$$
 [1]

$$
K = 2.3833. G
$$
 [2]

Where $\sigma'{}_{v}$ is the effective vertical stress in kPa. Minimum values of 3.1x10⁵ kPa and 7.4x10⁵ kPa are considered for shear and bulk moduli, respectively (James, 2009).

The bedrock material consists of a poor to medium quality rock mass, with properties based on values reported by Goodman (1989) and Wyllie and Mah (2004).

The different material properties for the static phase are presented in [Table 1.](#page-3-0) All materials were assumed to be homogeneous and isotropic except for the waste rock inclusions and dikes where the shear and bulk moduli varied with the vertical effective stress.

Once the static (equilibrium) total and effective stresses are defined, the constitutive models of the different zones are changed to simulate the dynamic behavior of the different materials. For the tailings, the PM4Sand model (Boulanger & Ziotopoulou, 2017) is used to capture their liquefiable behavior. The elastoplastic Mohr-Coulomb model with the hysteretic Sigmoidal model-sig3 option is assigned to waste rock inclusions, dikes, boundary element and starter dike zones. This Sigmoidal model can be expressed as (Itasca, 2016):

$$
G_{sec} = \frac{a}{1 + \exp\left(-\frac{(L - x_0)}{b}\right)}\tag{3}
$$

Where G_{sec} is the secant modulus, *L* is logarithmic strain, and *a*, *b*, and *x⁰* are non-dimensional parameters. The required parameters for this model are provided by Itasca (2016) for sands and clays. For waste rock, Ferdosi et al., (2015) calibrated the model based on the degradation curves of a gravely soils provided by Rollins et al. (1998); values of 1.02, -0.698 and -1.45, are used for, *a*, *b* and *x⁰* respectively.

The PM4Sand model is a critical-state compatible, stress-ratio based, bounding surface plasticity model developed for sand, which is based on the model presented by Manzari and Dafalias (1997) and extended by Dafalias and Manzari (2004). These constitutive equations are anchored in the Anisotropic Critical State Theory (ACST), but the PM4Sand model, developed and implemented by Boulanger & Ziotopoulou (2017), includes the evolution of the soil fabric based on plastic shear strains rather than plastic volumetric strains. The PM4Sand

model includes three primary parameters that need to be calibrated for each soil (or material) and 21 secondary parameters that can be adjusted from their default values to obtain a more material-specific calibration when sufficiently detailed information is available about their experimental behavior. In the present study, the calibration of the PM4Sand model for hard-rock tailings presented in Contreras (2021) is used; the primary and secondary parameters of the PM4Sand model were calibrated based on laboratory tests and field investigations. [Table 2](#page-3-1) presents the PM4Sand properties used for the tailings in the dynamic simulations.

Most non-linear constitutive models (such as PM4Sand) neglect hysteretic damping at low strains. As this aspect may be important, a viscous Rayleigh damping scheme is used here to introduce small strain damping. A value of 2% of stiffness and mass proportional Rayleigh damping centered on the mean frequency of the system is considered for the tailings. Elastic simulations are performed without damping to estimate the first mode of the system and the mean frequency of the model. The damping characteristics of the bedrock material were defined so that this layer does not amplify or attenuate the motion. A value of 3% of the stiffness and mass proportional Rayleigh damping centered on the mean frequency of the system is assigned to the bedrock material.

In addition, a nominal value of 0.2% of the stiffness and mass proportional Rayleigh damping with the mean frequency of the model is assigned to waste rock inclusions, dikes, boundary element and starter dike zones to control (eliminate) the effect of numerically generated noise on the simulations.

3 PARAMETRIC STUDY

As indicated above, the simulations conducted here serve to investigate the influence of the downstream slope and height of the impoundment on its seismic stability. The effect of these parameters is assessed by monitoring various indicators such as the critically displaced volume (*CDV*) of tailings, the deformation of the impoundment crest, and the permanent displacement of the downstream slope.

3.1 Geometry of the impoundments

In the parametric study, the downstream slope (V:H) varies between 1:7 and 1:12 and the height of the impoundment between 20 m and 50 m. The seismic response of the various models are compared to each other and the effect of each parameter is investigated individually by evaluating their influence on specific model response quantities.

3.2 Seismic hazard

The recording of the *M^w* 6.93 Loma Prieta (California) 1989 earthquake at station RSN810, located at a distance of 18.41 km from the surface projection of the rupture plan, is chosen as the input ground motion for the simulations presented here; other ground motions are considered in additional simulations presented by Zafarani (2021). This

recording is characterized by an average frequency of 6.5 Hz, a *PGA* of 0.3 g, an Arias Intensity, *AI*, of 1.2 m/s, and a significant duration (*D5-95*) of 13.72 s (PEER, 2019).

	Table 1. Constitutive model and material properties for the	
static phase simulations (see text for sources)		

¹ Equations [1] and [2]

Table 2. Calibrated PM4Sand model parameters for the tailings (Contreras, 2021)

Parameters	Hard rock tailings calibration
I _d -Density Index	0.6
G ₀ - Shear modulus Coefficient	502.5
h_{no} - Contraction rate parameter	0.55
e _{min} - Minimum void ratio	0.49
e_{max} - Maximum void ratio	1.1
n ^b - Bounding parameter	0.7
cz - Fabric growth parameter	150
$\omega'_{\rm cv}$ Critical state friction angle	35
Q- Critical state line parameter	12.7
R- Critical state line parameter	5.4

3.3 Waste rock inclusions configuration

The effect of waste rock inclusions configuration on the seismic response of tailings impoundments is also evaluated numerically using different width *W* (m) and edge-to-edge spacing *S* (m) of the inclusions. The width *W* of the inclusions varied between 12 and 25 m and spacing *S* between 55 and 155 m.

4 NUMERICAL RESULTS AND ANALYSIS

The effect of the downstream slope and height of the impoundments is quantified with two indicators: the critically displaced volume of tailings (*CDV*) and the average normalized horizontal displacement of the downstream slope (*SARx*).

Jibson (2011) reported that horizontal displacements exceeding 100 cm are likely to cause critical landslide movements. Similarly to the previous investigation of Ferdosi et al. (2015), the critically displaced volume (*CDV*) of tailings is defined here as the volume of tailings that displaced horizontally more than 100 cm due to the earthquake.

To further investigate the effect of the waste rock inclusion configuration on the behavior of the downstream slope of the model, the average value of the normalized horizontal displacement of all the nodal points along the downstream slope, *SARx*, is evaluated using the following equation:

$$
SAR_x = Average\left(\frac{X_{\text{disp}_i}}{H_i}\right)
$$
 [4]

Where X_{disp_i} is the horizontal displacement of each nodal point along the downstream slope at the end of shaking, and H_i is the initial height (elevation) of each nodal point above the bedrock layer.

4.1 Influence of the downstream slope

The maximum horizontal displacement is computed under the Loma Prieta ground motion to assess the effect of the downstream slope on the seismic behavior of the unreinforced impoundment model. [Figure 2](#page-4-0) shows the contours of the maximum horizontal displacements of three unreinforced models with a height of 40 m and downstream slopes of (a) 1:8, (b) 1:10, and (c) 1:12 at the end of the ground motion. In addition to these results, it is noted that the same ground motion causes 75% of the impoundment volume of the model with a height of 40 m and downstream slope of 1:7 to displace critically after 17 seconds. This simulation then stopped because of the excessive deformation of at least one element and the inability of the mathematical formulation to account for such large strain; hence, the results for this simulation are not show in [Figure](#page-4-0) [2.](#page-4-0)

The simulation results indicate that, as expected, a decrease of the downstream slope of the impoundment leads to a decrease in *CDV*. For instance the seismic response of the impoundment with downstream slope of 1:12 [\(Figure 2](#page-4-0) (c)) shows that 58% of the model's volume is displaced more than 100 cm at the end of the motion, compared with more than 70 % for a slope of 1:8. These percentages are significant and would likely lead to a failure of the unreinforced impoundments.

Figure 2. Contours of the horizontal displacements of the unreinforced impoundment models with a height of 40 m and an external slope of (a) 1:8, (b) 1:10, and (c) 1:12

The effect of adding waste rock inclusions on the seismic behavior of the impoundment is illustrated in [Figure 3](#page-4-1) for the models with slopes of (a) 1:7, (b) 1:8, (c) 1:10, and (d) 1:12 and height of 40 m, using three 16-m wide inclusions under the Loma Prieta earthquake loading. Note that the spacing between the inclusions depends on the geometry of the impoundment, its height and downstream slope. The four configurations presented in [Figure 3](#page-4-1) are nonetheless comparable in spite of small spacing differences for the inclusions. As seen in [Figure 3](#page-4-1) (a), 35% of the reinforced impoundment with a slope of 1:7 is displaced critically at the end of the motion compared to 75% for the case without inclusion. Comparing the seismic response of the reinforced and non-reinforced models shows that the volume critically displaced is decreased by a factor of 4.28 for the external dike with a slope of 1:10 and 5.8 for a downstream slope of 1:12. These results tend to indicate that the use of waste rock inclusions proportionally impact more significantly the volume displaced by seismic loading for shallower downstream slopes (i.e., 1:10 and 1:12 here).

The average value of the normalized horizontal displacement along the downstream slope (*SARx*) was computed to evaluate the effect of the inclusions configuration on the behavior of the impoundment. [Figure](#page-4-2) [4](#page-4-2) illustrates the variation of the *SAR^x* as a function of a nondimensional geometric parameter *Wvs* defined by:

$$
W_{\rm vs} = V_{\rm r} \cdot (1 - \frac{S_{\rm 1}}{S_{\rm max}})
$$
 [5]

where V_r is the volume ratio of the WRI and impoundment $(\frac{\text{volume of the two number of two times}}{\text{Volume of the impoundment under the slope}}), S_1$ is the spacing Volume of WRI between the starter dike and closest inclusion, and S_{max} is the spacing between the starter dike and the furthest waste rock inclusion in the model.

Figure 3. Final contours of horizontal displacements of reinforced impoundment models for WRI with a width of 16 m, a height of 40 m, and (a) spacing of 60 m, slope 1:7 (b) spacing 70 m, slope 1:8 (c) spacing 90 m, slope 1:10 (d) spacing 121 m, slope 1:12

Simulations results are presented for four downstream slopes (i.e., 1:7, 1:8, 1:10, and 1:12), with global trendlines. As seen in [Figure 4,](#page-4-2) the results indicate that an increase in the volume of inclusions in the impoundment tends to decrease the value of *SARx*. These illustrate how the width and spacing of the waste rock inclusions influence the seismic response of the tailings impoundment, when expressed in terms of shear strain and horizontal displacement of the downstream slope. The effect on the value of *SAR^x* for shallower downstream slopes (i.e., 1:12 and 1:10) is limited, while it is much more pronounced for steeper slopes (i.e. 1:8, 1:7). In the latter cases, the *SAR^x* values are significantly greater for the impoundment models with the lowest value of W_{ns} .

Figure 4. Variation of the *SAR^x* value as a function of parameter *Wvs* for the reinforced impoundment models with a height of 40 m and downstream slopes of 1:7, 1:8, 1:10, and 1:12

4.2 Effect of height

The effect of the height of the dikes confining the impoundment on the *CDV* of the various reinforced tailings impoundments under the imposed ground motion is illustrated in [Figure 5](#page-5-0) (results with much larger *CDV* obtained for unreinforced impoundments are not shown here). The legends (inserted captions) in Figure 5 gives the configuration of the waste rock inclusions; the first number is the width of the inclusions, the second number is the spacing between the inclusions, the third number is the height of the impoundment, and the fourth number gives the downstream slope of the external dikes. The three configurations are essentially equivalent, although the spacing differs slightly as indicated above.

The results indicate that, as expected, the volume displaced by more than 100 cm for an impoundment with a height of 40 m [\(Figure 5](#page-5-0) a) is smaller than for a height of 50 m [\(Figure 5](#page-5-0) b). The critically displaced volume for the 40 m high impoundments is limited to the zones under the downstream slope. However, for configurations 12-70-50- 8H and 16-70-50-8H, with a height of 50 m, the critically displaced volume extends beyond the zone under the downstream slope.

For configuration 12-80-40-8H , the entire zone under the downstream slope (45 % of the total impoundment volume) is displaced by more than 100 cm. Increasing the width of the inclusions from 12 m to 16 m reduces the critically displaced volume, which is then limited to the top half of the downstream slope (29% of the total volume). Further increasing the width of the inclusions to 20 m results in an even lower critically displaced volume (i.e., 9%).

The critically displaced contours for configuration 12- 70-50-8H in [Figure 5](#page-5-0) (b) show that 60% of the total impoundment volume is displaced by more than 100 cm at the end of the motion. Increasing the width of the inclusions from 12 m to 16 m (16-70-50-8H) causes only a 5% reduction in the critically displaced volume, which is less significant compared to the models with a height of 40 m. However, by further increasing the width of the inclusion to 20 m (20-65-50-8H), the critically displaced volume is reduced by a factor of 2.6 compared to configuration 16- 70-50-8H. These results indicate that wider inclusions tend to reduce the displacements, but this effect appears to be less pronounced for the thicker tailings impoundment.

[Figure 6](#page-5-1) shows the variation of the *SAR^x* versus *Wvs* for four different heights of the impoundment (i.e., 20 m, 30 m, 40 m, and 50 m) with a downstream slope of 1:8 and various WRI configurations. The *SAR^x* values obtained for the models with heights of 20 m and 30 m are relatively constant, due to the small horizontal displacements. Increasing the height of the model to 50 m leads to larger displacements, with *SAR^x* increasing by a factor of 6 compared with the lowest reinforcement. The variations of *SAR^x* shown in Figure 6 also indicate that the tailings height has a larger impact on the seismic response of the impoundment when compared with the downstream slope (for the range of values considered here).

Figure 5. Volume of the critically displaced tailings within the reinforced impoundments at the end of the ground motion, for a slope of 1:8, and a height (a) of 40 m, and (b) 50 m; the WRI and dikes configurations are given in the inserted captions: the first number is the width of the inclusions, the second is their spacing, the third is the height of the impoundment, and the fourth gives the downstream slope of the external dikes.

Figure 6. Variation of the *SAR^x* value as a function of *Wvs* for the reinforced impoundment models with a downstream slope of 1:8 and heights of 20 m,30 m, 40 m, and 50 m

5 DISCUSSION AND CONCLUSION

A parametric study of the seismic behavior of tailings impoundments, with some reinforced with waste rock inclusions, has been performed based on dynamic numerical analysis conducted with FLAC Version 8.00 (Itasca, 2016). The simulations focus on impoundments

with dikes built using the upstream method. The PM4Sand model is used to simulate the tailings behavior, and the Mohr-Coulomb model represents the WRI and other materials. The effects of the downstream slope and height of the impoundment on the seismic stability were investigated, with four downstream slopes, 1:7, 1:8, 1:10, and 1:12, and four heights of the dikes 20 m, 30 m, 40 m, and 50 m. The influence of the downstream slope and height of the impoundment is quantified with two indicators: the critically displaced volume of tailings (*CDV*) and the average normalized horizontal displacement of the downstream slope (*SARx*).

Results of the simulations for the four inclinations of the downstream slope show that an increased volume of inclusions in the impoundment leads to a decrease in the value of *SARx*. This effect is more significant for the steeper slopes of 1:8 and 1:7.

The effect of height follows a similar trend, with higher impoundments producing larger volumes of critically displaced tailings. For instance, the critically displaced volume for the 40 m high configurations is limited to the area under the downstream slope, but most configurations with a height of 50 m lead to critically displaced volumes extending beyond the crest. Increasing the width of the inclusions in the impoundments decreases the *CDV* for a height of 40 m. The effect of wider inclusions appears less pronounced for impoundments with a height of 50 m, which expedient larger deformations and excess pore water pressure ratios.

The results of the various dynamic simulations indicate that tailings thickness has a major impact on the seismic response of the impoundment. This factor appears more significant than the downstream slope for the cases analyzed here.

Ongoing work focuses on the development of design guidelines, with further simulations are being performed. In these simulations, different relative densities of the tailings are considered, and a variety of input ground motions are applied to assess the effect of seismic hazard on the behavior of the impoundments

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