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Prediction of ground motion parameters in 1D ground response analysis

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ABSTRACT

Ground motion parameters are used in various simplified seismic design procedures to represent the seismic hazard, such as the assessment of liquefaction, seismic slope stability, structural response, and the estimation of the damage potential of earthquakes. In practice, these can be obtained from the results of ground response analyses. However, their assessment resulting from these procedures has not been widely investigated to date. In this study, equivalent-linear (EQL) and nonlinear (NL) 1D ground response analyses are conducted for a dataset of 5 sites and 89 ground motion recordings. The uncertainty in the prediction of 10 commonly used ground motion parameters evaluated at the ground surface is quantified and compared. The findings of this study reveal a tendency towards the over-prediction of most of these parameters. It was found that one of the parameters, the mean period T_m, yielded the lowest bias and variability compared to the other measures.

RÉSUMÉ

Plusieurs procédures de conception sismique, telles que des analyses du potentiel de liquéfaction, de la stabilité des pentes, de la réponse structurale ou de l’estimation du potentiel de dommages sismiques, utilisent certains paramètres de mouvements de sols pour représenter l’aléa sismique. En pratique, il est commun d’utiliser des analyses de réponse de dépôts de sols pour la détermination de ces paramètres. Or, à ce jour, peu d’études ont toutefois été recensées à ces fins. Dans cette étude, des analyses 1D de réponse de sols équivalente-linéaires (EQL) et nonlinéaires (NL) sont réalisées pour un ensemble de 5 sites et 89 enregistrements de séismes. L’incertitude reliée à la prédiction de 10 paramètres communément utilisés en pratique et évalués à la surface du dépôt de sol est quantifiée et comparée. Les résultats révèlent une tendance générale vers la sur-prédiction des paramètres étudiés. La période moyenne T_m démontre la plus faible incertitude et variabilité en comparaison aux autres paramètres.

1 INTRODUCTION

In practice seismic hazard is most commonly described using period-dependent spectral accelerations, such as provided in the form of seismic hazard maps by the National Building Code of Canada (NBCC, 2015). These are defined to represent the seismic response of one-degree-of-freedom (*SDOF*) oscillators at different natural frequencies. Other intensity measures, such as the peak ground acceleration (*PGA*), are also predominantly used, for example, in the definition of the cyclic stress ratio (*CSR*) integrated into liquefaction evaluation procedures (e.g. Youd et *al*., 2001; Boulanger and Idriss, 2014). However, several studies have demonstrated that the use of peak parameters may be limited in representing the characteristics of a ground motion as it does not exhaustively represent its intensity, duration, and frequency content (Kramer, 1996). Thus, other parameters have been proposed to describe these features and have been integrated into simplified seismic design procedures to estimate liquefaction triggering, seismic slope stability, potential for earthquake damages, etc.

In practice, the seismic hazard can be estimated on the rock from seismic hazard maps or through site-specific probabilistic seismic hazard analysis, and then modified to account for soil effects using 1D ground response analysis. The latter evaluate the propagation of seismic waves in a soil deposit as a function of soil and bedrock properties. These procedures are essential since the presence of soft soils can lead to the amplification or attenuation of the motion upon propagation, which can significantly influence the seismic hazard.

Initial solutions to ground motion propagation problems are centered on the use of unidimensional (1D) equivalent-linear (EQL) ground response analyses, such as included in the SHAKE software (Schnabel et *al*., 1972). However, recent studies have underlined the limitation of this approach in modeling the true nonlinear behavior of soils. In particular, difficulties in modeling motions inducing shear strains greater than 0.05% (Kaklamanos et *al*., 2015), have been highlighted. In recent years, an increase in the number of studies related to the evaluation of nonlinear models has been observed (e.g. Hashash et *al*., 2010; Kim and Hashash, 2013; Kaklamanos et *al*., 2013; Kaklamanos et *al*., 2015; Shi and Asimaki, 2017; Kaklamanos and Bradley, 2018; Li et *al*., 2018), and its use in practice has increased. So far, however, controlled studies on the estimation of ground response parameters from the results of nonlinear ground response analysis have been relatively scanty. It is, thus, essential to assess the corresponding uncertainty associated with the use of ground motion parameters for an appropriate use in alternate design procedures.

The purpose of this study is to evaluate and validate the precision of the prediction of commonly used ground motion parameters from the results of ground response analyses. EQL and NL ground response analyses are thus performed using the software DEEPSOIL. A total of 10 ground motion parameters are investigated: Arias Intensity (*AI)*, significant durations ($D_{5\text{-}95}$ & $D_{25\text{-}75}$), number of equivalent loading cycles (N_{eq}), Cumulative Absolute Velocities (*CAV* & CAV_{STD}), Shaking Intensity Rate (*SIR*), and characteristic periods (T_m, T_o, or T_p). The analyses are performed for a dataset of 5 sites, including 4 downhole seismic arrays and one centrifuge test. From these sites, 89 ground motion recordings are selected to represent both low and high strain cyclic loading conditions.

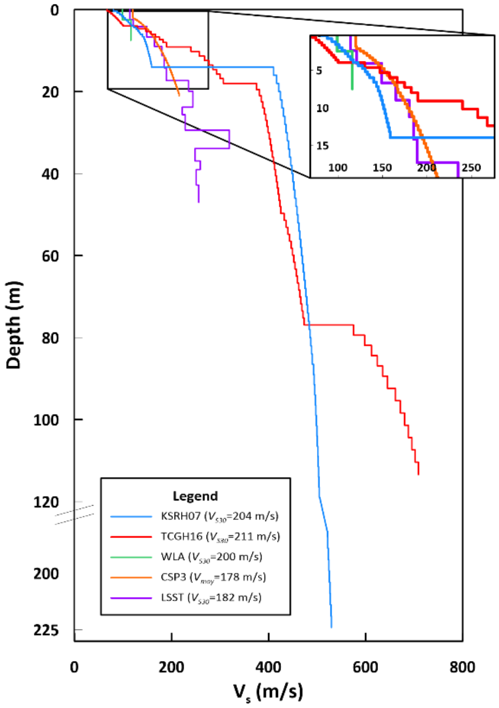

Figure 1. Shear wave velocity profiles of the 5 selected sites.

In the following, the dataset of sites and ground motions is described, followed by the presentation of the adopted methodology for the selection of ground motion parameters, presented in Table 1, and the quantification of bias and variability. The results are first presented from the evaluation of response spectrum (*PSA*) residuals which are compared against the shear strain index (I_γ) (Idriss, 2011) to identify any occurring biases. A similar example is provided for one parameter of interest, the Arias Intensity (*AI*). Finally, the residuals for all ground motion parameters are evaluated and compared against each other.

3 SITE AND GROUND MOTION SELECTION

In this study, sites that can be accurately modeled by 1D ground response analyses are selected. Thus, to verify the absence of 3D stratigraphy effects, the procedure proposed in Thompson et *al*. (2012) is applied herein. This method compares the variability between empirical transfer functions and their goodness-of-fit to elastic analytical solutions, obtained from Kramer et *al*. (1996), at each site. Based on the results, the 1D hypothesis was deemed appropriate to model the selected sites.

The dataset is composed of 5 sites including two Kiban-Kyoshin (KiK-net) network stations (KSRH07 and TCGH16) located in Japan, the Wildlife Liquefaction Array (WLA) in California, and the Lotung Array (LSST) in Taiwan. In addition, one centrifuge experiment from Wilson et *al*. (1998) is selected. This test is selected because the stratigraphy of the model and the testing conditions are representative of a 1D wave propagation problem.

The shear-wave velocity (V_s) profiles of the five sites and corresponding values of the average V_s in the upper 30 m (V_{S30}) are presented in Figure 1. Based on these values, most sites are characterized as seismic site class D (NEHRP, 1993) and CSP3 as site class E.

From these sites, 89 ground motion recordings are selected to represent varying nonlinearity and ground

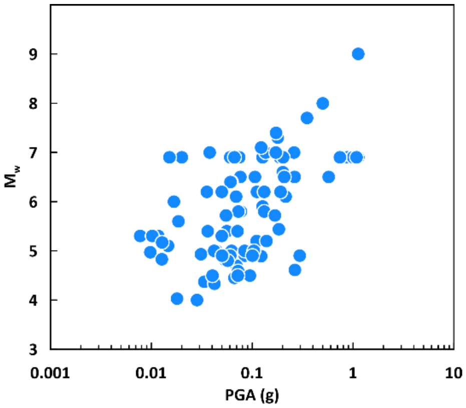

Figure 2. Distribution of the magnitude M_w versus the surface peak ground acceleration (PGA) for the 89 motions.

Table 1. Selected ground motion parameters

Parameter	Definition	Reference
Arias Intensity (*AI*)	$\frac{\pi}{2g}\int_0^{t_{max}} a(t)^2 dt$	Arias (1970)
Significant duration (D_{5-95} & D_{25-75})	Time interval between 5 and 95% or 25 and 75% of the total *AI*	Trifunac and Brady (1975)
Number of equivalent loading cycles (N_{eq})	See reference, computed for a reference acceleration of $0.65 \times PGA$	Seed et *al*. (1975)
Cumulative absolute velocity (*CAV*)	$\int_0^{t_{max}} \lvert a(t) \rvert dt$	EPRI (1988)
Standardized cumulative absolute velocity (CAV_{STD})	$\sum_{i=1}^{N}(H(PGA_i - 0.025)\int_{t=t_i}^{t_{i+1}} \lvert a(t) \rvert dt$ where *H(x)* is the Heaviside step function	Kramer and Mitchell (2006)
Shaking Intensity Rate (*SIR*)	$\frac{AI_{5-75}}{D_{5-75}}$	Dashti et *al*. (2010)
Mean period (T_m)	$\sum_i {C_i}^2 \left(\frac{1}{f_i}\right) / \sum_i {C_i}^2$ where C_i are the discrete Fourier amplitude coefficients and f_i are the associated discrete Fourier transform frequencies between 0.25 and 20 Hz	Rathje et *al*. (1998)
Smoothed spectral predominant period (T_o)	$\sum_i T_i \cdot ln\left(\frac{S_a(T_i)}{PGA}\right) / \sum_i ln\left(\frac{S_a(T_i)}{PGA}\right)$ where *Ti* are logarithmically spaced discrete periods with $S_a/PGA \geq 1.2$ and $S_a(T_i)$ are spectral accelerations.	Rathje et *al*. (1998)
Predominant spectral period (T_p)	$\max(S_a(T))$	

motion solicitation levels. The ground motion distribution of moment magnitudes (M_w) versus peak ground surface acceleration (*PGA)* is presented in Figure 2, with values ranging from 4.0 to 9.0 and from 0.008 to 1.13 g, respectively.

4 METHODOLOGY

4.1 Ground response analysis procedures

Equivalent-linear (EQL) and nonlinear (NL) unidimensional ground response analyses are conducted using the software DEEPSOIL (Hashash et *al*., 2016). The target modulus reduction and damping curves of Zhang et *al*. (2005) for soils and Choi (2008) for rock are used herein. The cyclic behavior of soils is controlled using the General Quadratic/Hyperbolic model (GQ/H) with shear strength correction (Groholski et *al*., 2016), defined by the Mohr-Coulomb criterion for granular soils and the Ladd and Foott (1974) relationship for cohesive soils. Frequency independent small-strain damping and MRDF-UIUC unload-reload rules are employed as recommended by Phillips and Hashash (2009). The bottom boundary condition is defined as rigid since the input motions were recorded as “within” motions. Finally, a bandpass filter and a baseline correction are applied to all recorded motions and the ground response analysis results. Note that for motions where both orthogonal components of the motion were recorded, ground response analyses are first conducted individually for each component, and the geometric mean is used to compute the combined spectral acceleration.

4.2 Selected ground motion parameters

In this study, the following ground motion parameters are investigated: *AI*, D_{5-95} and D_{25-75}, N_{eq}, *CAV*, CAV_{STD}, *SIR*, T_m, T_o, T_p. Table 1 presents a summary of the parameters along with their definitions and references. These parameters are selected to represent various components of ground motion characteristics regarding its intensity, duration, and frequency content. Most of these parameters are also integrated in commonly used simplified seismic design procedures found in practice. For example, procedures for the assessment of liquefaction (Kayen and Mitchell, 1997; Kramer and Mitchell, 2006), potential for earthquake damages (Forschaar et *al*., 2012) structural response (Dommer et *al*., 2004), or seismic slope stability (Chousianitis et *al*., 2014) use some of the aforementioned parameters. Note that the ground motion parameters are computed from the predicted surface acceleration time series obtained from the ground response analyses results and are compared with the observed motion.

4.3 Quantification of uncertainty

The accuracy of the predictions of EQL and NL ground response simulations is quantified by computing the residuals between the observed and predicted value of a given ground motion parameter (*GMP*) as described in Equation 1.

$$GMP_{res} = \ln(GMP_{obs}) - \ln(GMP_{pred}) \qquad [1]$$

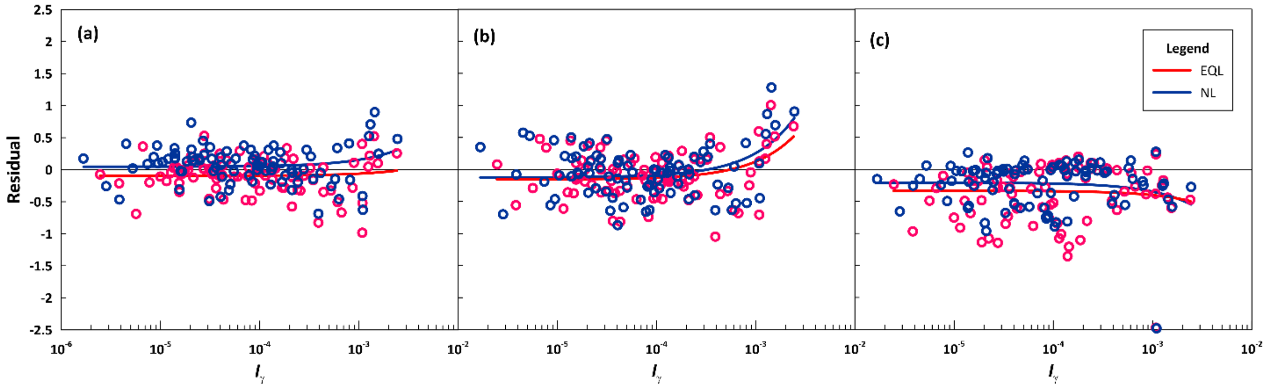

Figure 3: (a) $PSA_{T=0.01s}$, (b) $PSA_{T=0.1s}$, and (c) $PSA_{T=1.0s}$ residuals as a function of the shear strain index I_γ for EQL and NL analyses.

Based on this equation, positive residuals are associated with an under-estimation of the parameter while a negative residuals represents an over-prediction. The computed residuals are subsequently compared against the shear strain index I_γ proposed by Idriss (2011), defined by Equation 2.

$$I_\gamma = \frac{PGV_{dowhole}}{V_{S30}} \qquad [2]$$

This ratio is chosen since it is independent of the modeling approach as it can be defined prior to the analysis. Shi and Asimaki (2018) also demonstrated its reliability in being used as a proxy for the computed maximum soil shear strains (γ_{max}).

5 RESULTS AND DISCUSSION

In the following, the accuracy of the ground response analyses is first evaluated in terms of spectral acceleration residuals. Then, an example of the analysis of the residuals is provided for one of the parameters, the Arias Intensity (*AI*). Finally, the predictions of all parameters are evaluated and compared against each other and parameters with the most optimal performance, or lowest bias, are identified.

5.1 Spectral accelerations

The performance of ground response models is typically described by examining the residuals of pseudo-spectral accelerations at characteristic periods ($PSA(T_i)$). As mentioned previously, these are also one of the main ground motion parameters used in design. In Figure 3 are presented the computed residuals at periods T = 0.01, 0.1, and 1.0 s for the EQL and NL analyses as a function of the shear strain index I_γ. Note that a linear fit is added to evaluate trends in the data.

The results show that $PSA_{T=0.01s}$ and $PSA_{T=0.1s}$ ranges from -1.04 to 1.28 approximatively. The trends appear to be mostly constant, except for the higher strain levels, where a slightly upward tendency, which is more pronounced for $PSA_{T=0.1s}$, is observed. The residuals for $PSA_{T=1.0s}$ are shown to be more negative across all the levels of nonlinearity with values ranging from -1.35 to 0.28, which indicates the tendency of the over-prediction of the motion compared to other periods. In particular, higher residuals are obtained for the EQL analysis as opposed to the NL analysis. This corroborates the results of previous studies (e.g. Kwok et *al*., 2008; Kim and Hashash, 2013; Kaklamanos et *al*., 2015) wherein similar results were found. Thus, the results in Figure 3 underline the ability of the ground response models in modeling the observed response on the basis of the response spectra evaluation since no significant bias was identified.

5.2 Arias intensity

The Arias Intensity (*AI*) is obtained by integrating the squared acceleration time histories of a ground motion, as defined in Table 1. It aims to represent the cumulative energy of a ground motion as a function of time. This quantity has been historically used for many applications for the assessment of liquefaction (e.g. Kayen and Mitchell, 1997; Kramer and Mitchell, 2006), seismic slope stability (e.g. Harp and Wilson, 1995; Jibson et *al*.,

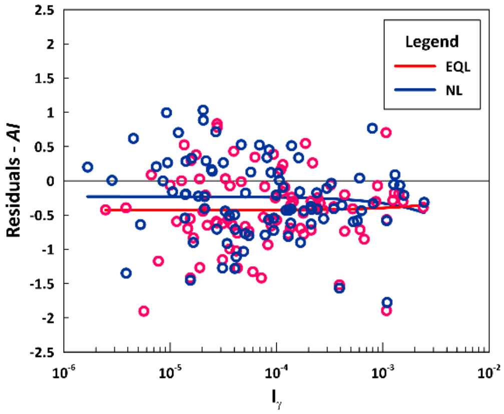

Figure 4: Arias Intensity residuals as a function of the shear strain index I_γ for EQL and NL analyses.

2000; Hsieh and Lee, 2011; Chousianitis et al., 2014), and structural response (e.g. Cabanas et al., 1997).

In Figure 4 are presented the residuals of *AI* for the EQL and NL analyses. Note that when both orthogonal components of the motion are available, *AI* is computed as the sum of the Arias Intensities of each of the individual components. In general, the models follow a tendency of over-predicting *AI* across all strain levels. For low to moderate strain levels, an improvement is obtained by the use of the NL analysis in comparison to the EQL analysis.

These findings were found to be consistent with the results reported in Hashash et *al*. (2015). In this study, the prediction of *AI* from the results EQL and NL ground response analyses were compared with centrifuge test results for 6 large ground motions (M_w > 6.7 and *PGA* > 0.33 g). The authors found that the analysis of the residuals demonstrated a similar tendency towards the over-prediction of this parameter and a higher precision for the NL analysis.

5.3 Summary

In this section, a summary of the comparison of the precision and accuracy of the prediction of all the ground motion parameters is provided. The residuals obtained from the spectral acceleration evaluation are included for direct comparison. In Figure 5 are presented the range of residuals for each ground motion parameter for both EQL and NL analyses. Note that the box limits correspond to the 25th and 75th percentile of the dataset, whiskers limits are defined as 1.5 of the interquartile range (IQR), and the median is identified by a vertical line in the IQR.

The results reveal that, in general, increased variability in the residuals is obtained for the investigated parameters when compared to the residuals from the *PSA* spectrum presented previously in Figure 3. From these results, a tendency towards the over-prediction of these parameters is revealed except for *SIR* and T_m for the NL analysis, which are slightly under-predicted. The range of residuals appear to be the largest for parameters *SIR*, $D_{25\text{-}75}$, and T_p. The largest median error is also committed for parameter *AI* for the EQL analysis and $D_{25\text{-}75}$ for the NL analysis.

Overall, the lowest uncertainty and variability is obtained for T_m, with average residuals yielding a value close to zero. This is explained by the fact that T_m is defined directly from the Fourier Amplitude Spectrum (*FAS*), which is more representative of the frequency content of a motion as opposed to *PSA*. In fact, the latter consists of an approximation of the response of SDOF oscillators, as mentioned previously. Thus, this confirms previous work by Rathje et *al*. (2004) which showed that T_m was subjected to less variability than other characteristic periods. Excluding T_m, the parameters N_{eq} and *CAV* also demonstrate low variability and range of residuals compared to the other parameters. In general, the range of residual obtained from the nonlinear analysis was revealed to be slightly narrower than for the equivalent-linear analysis across all the studied parameters.

6 CONCLUSION

The purpose of the present study was to evaluate the accuracy and variability obtained when predicting common ground motion parameters from the results of 1D ground response analyses. These metrics have a significant role in practice as they are often integrated in simplified seismic design procedures. These ground motion parameters also ought to represent one or more components of the amplitude, duration or frequency

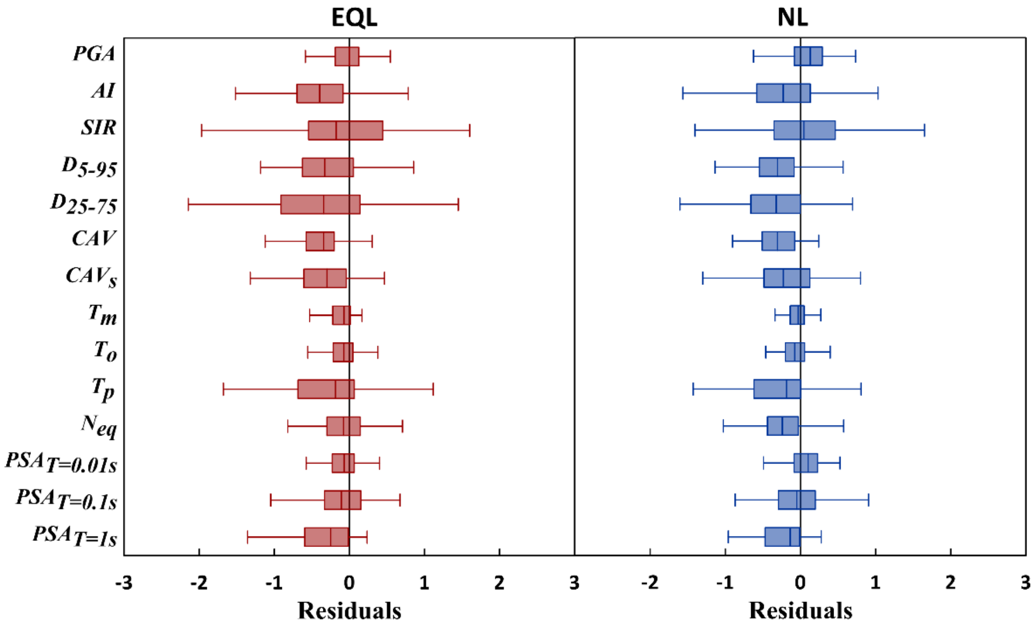

Figure 5. Distribution of residuals per ground motion parameter for the EQL and NL analyses

content of a ground motion. Thus, these have been shown to be more reliable than the use of peak parameters such as the *PGA*. Few studies have attempted to evaluate these measures from the results of ground response analyses and has been the motivation behind the present study.

In this paper, EQL and NL 1D ground response analyses are conducted for a total of 5 sites and 89 ground motion recordings. This selection was carefully achieved in order to respect 1D assumptions and include motions with varying amplitudes and nonlinearity levels. Overall, 10 ground motion parameters commonly used in practice are selected and investigated. The analysis of the residuals between the observed and predicted motion is then presented to quantify the bias and variability.

The main findings of this study suggest that significant bias and uncertainty occurs when evaluating the investigated ground motion parameters from the results of EQL and NL 1D ground response analysis. A noticeable tendency towards the over-prediction of most of these measures was revealed. In general, this bias unveils a divergence from the analysis of the *PSA* residuals, where the range of residuals was found to be generally lower. Therefore, these results imply that while the response spectrum is accurately captured by 1D ground response analysis, it is not sufficient to represent the full extent of the seismic hazard, since components of intensity and duration of a ground motion are not as accurately predicted. Moreover, this research has shown that one of the parameters, the mean period T_m, has an overall increased performance over the other parameters, since it exhibits a significant decrease in both uncertainty and variability. It is worth noting that the precision was found to be higher than *PSA* residuals. In addition, the predictions of parameters *PGA*, *CAV*, N_{eq}, and T_o also showed some improvement over the other quantities.

Overall, important conclusions from this research demonstrate the difficulties of using ground response analysis to evaluate ground motion parameters. Therefore, this work emphasizes the importance of exercising caution when using seismic design procedures based on these parameters in order to avoid increasing the uncertainty of these alternate procedures. Thus, this work helps identify adequate ground motion measures for this purpose that aims to offer an accurate representation of the seismic hazard.

Further research is currently being undertaken on this topic as we are planning to expand our dataset by integrating more sites, ground motion recordings, and ground motion parameters.

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