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EFFECT OF SPECIMEN SIZE ON NORMALIZED DISSIPATED ENERGY PER UNIT VOLUME

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ABSTRACT

The quantity of energy dissipated in a unit volume of soil during cyclic loading can be used as a measure of the soil's ability to withstand liquefaction. This energy is referred to as the normalized dissipated energy per unit volume (NDEPUV). The greater the NDEPUV required to induce liquefaction, the more seismic energy must be input into the soil during an earthquake for liquefaction to occur. The NDEPUV for a soil subjected to a seismic event or a laboratory test can be calculated from the stress-strain behavior of the soil.

In this study, the effect of specimen size on NDEPUV was examined using strain-controlled cyclic triaxial tests performed on specimens of uniform sand prepared to a relative density of 40%. Four specimens were tested at each of four volumes and the NDEPUV required to induce liquefaction was determined. The specimen volumes ranged from 87 to 1525 cubic centimeters. In addition to NDEPUV, the number of cycles of loading required to trigger liquefaction and the pseudo energy capacity (the calibration parameter for the GMP pore pressure model) were examined.

The NDEPUV was found to be independent of the specimen volume for three of the four specimen sizes. The largest specimens were found to require less energy per unit volume to liquefy. Conversely, the number of cycles to trigger liquefaction and the pseudo energy capacity were both found to be independent of specimen volume for all four specimen sizes.

RÉSUMÉ

La quantité d'énergie dissipée dans un volume unitaire de sol pendant le chargement cyclique peut être utilisée comme mesure de la capacité du sol à résister à la liquéfaction. Cette énergie est appelée énergie dissipée normalisée par unité de volume (NDEPUV). Plus le NDEPUV est important pour induire la liquéfaction, plus l'énergie sismique doit être introduite dans le sol pendant un tremblement de terre pour que la liquéfaction se produise. Le NDEPUV pour un sol soumis à un événement sismique ou à un essai en laboratoire peut être calculé à partir du comportement contrainte-déformation du sol.

Dans cette étude, l'effet de la taille des échantillons sur le NDEPUV a été examiné à l'aide de tests triaxiaux cycliques à déformation contrôlée effectués sur des échantillons de sable uniforme préparés à une densité relative de 40%. Quatre échantillons ont été testés à chacun des quatre volumes et le NDEPUV requis pour induire la liquéfaction a été déterminé. Les volumes des spécimens variaient de 87 à 1 525 centimètres cubes. En plus du NDEPUV, le nombre de cycles de chargement requis pour déclencher la liquéfaction et la capacité pseudo-énergétique (le paramètre d'étalonnage pour le modèle de pression interstitielle GMP) ont été examinés.

Le NDEPUV s'est révélé indépendant du volume de l'échantillon pour trois des quatre tailles d'échantillon. Les plus gros spécimens se sont avérés nécessiter moins d'énergie par unité de volume pour se liquéfier. Inversement, le nombre de cycles pour déclencher la liquéfaction et la capacité pseudo-énergétique se sont avérés indépendants du volume de l'échantillon pour les quatre tailles d'échantillons.

1 INTRODUCTION

Since the introduction of Seed and Idriss's (1971) stress–based method of liquefaction analysis in 1971, it has been the primary means by which engineers have evaluated the liquefaction potential of a soil in the field. Beginning in the 1980's alternative methods of liquefaction analysis based on the energy dissipated in the soil have been developed. If the normalized dissipated energy per unit volume required to initiate liquefaction (NDEPUV) in a soil is known, computer models can be used to create estimates of the soil's liquefaction potential based upon its predicted stress-strain behavior.

Normalized dissipated energy per unit volume required to induce liquefaction is typically determined through laboratory testing of reconstituted specimens. While a number of different laboratory tests can be used, the most commonly used test for this purpose is the cyclic triaxial test. When performing cyclic triaxial tests to determine the normalized dissipated energy per unit volume required to induce liquefaction, a variety of sample sizes have been used. While the energy dissipated is based on a unit volume, it remains to be determined if specimen volume has any effect upon the amount of energy that must be dissipated in the specimen in order to trigger liquefaction.

In order to study the effects of specimen size, the authors performed cyclic triaxial tests on four sets of specimens with volumes ranging from 69 to 1525 cubic centimeters. This paper will first review the basics of energy-based liquefaction analyses and then provide the details of the study performed. It will then report the results and finally presents the conclusions derived from the study.

2 BACKGROUND ON DISSIPATED ENERGY AND PORE PRESSURE GENERATION

In addition to the more common stress-based approach (Seed and Idriss 1971; Youd et al. 2001), liquefaction susceptibility analyses can be performed using an energy-based approach. In an energy-based approach, the energy demand is quantified by the normalized dissipated energy per unit volume imparted by the earthquake, while the energy capacity is quantified by the normalized dissipated energy per unit volume required to initiate liquefaction in the soil. The demand is a function of the stress-strain behavior of the soil under the assumed seismic loading and the capacity can be determined through laboratory tests such as a cyclic triaxial test.

The normalized dissipated energy per unit volume, W_s, is the energy dissipated per unit volume of soil divided by the initial effective confining pressure. For cyclic triaxial loadings, W_s, the normalized dissipated energy per unit volume required to induce liquefaction can be calculated by Equation 1 (Green 2001):

$$W_s = \frac{1}{2\sigma'_o}\sum_{i=1}^{n-1}(\sigma_{i+1}+\sigma_i)(\varepsilon_{i+1}-\varepsilon_i) \qquad [1]$$

Where: W_s is the dissipated energy normalized by the initial mean effective confining stress; σ'_o is the initial mean effective confining stress; n is the number of load increments applied to the specimen in order to initiate liquefaction; σ_i and σ_{i+1} are the applied shear stresses at load increment i and i+1, respectively; and ε_i and ε_{i+1} are the shear strains at load increment i and i+1, respectively.

The GMP model (Green et al 2000) relates the normalized unit energy, W_s, to the pore pressure ratio, r_u. The pore pressure ratio is the ratio of the excess pore water pressure generated during cyclic loading to the initial effective confining stress. When the pore pressure ratio reaches unity, the soil is considered to have liquefied. The formulation of the GMP model is provided in Equation 2 (Green et al 2000):

$$r_u = \sqrt{\frac{W_s}{PEC}} \le 1 \qquad [2]$$

where: W_s is the energy dissipated per unit volume of soil divided by the initial effective confining pressure and PEC is the pseudo energy capacity, a calibration parameter. A soil's pseudo energy capacity can be determined using either cyclic test data or correlations.

3 LABORATORY TESTING PROGRAM

For this study, 16 strain-controlled cyclic triaxial tests were performed on specimens of Ottawa C-109 sand prepared to a relative density of 40%. These specimens were tested in groups of four with one of four specimen volumes: 3.4 cm by 7.6 cm (volume of 69 cm^3), 5.1 cm by 10.2 cm (volume of 206 cm^3), 7.1 cm by 15.4 cm (volume of 610 cm^3) and 10.2 cm by 18.8 cm (volume of 1525 cm^3).

The first phase of the study consisted of performing index testing on the sand used in the study in order to quantify soil properties and obtain parameters necessary for the later testing. The sand used in the testing program was Ottawa C-109 sand, a commercially-produced silica sand from Illinois. Ottawa C-109 sand is a poorly-graded, medium to fine sand, with a mean grain size, D_{50}, of 0.33 mm. Its grain-size properties are 100 percent passing the No. 20 sieve (0.84 mm), 0 percent passing the No. 200 sieve, a coefficient of uniformity, C_u, of 2.18 and a coefficient of curvature, C_c, of 1.34. The shape of the grains vary from sub-rounded to rounded. It has a maximum index void ratio of 0.688, a minimum index void ratio of 0.436 and a specific gravity of 2.65.

The second phase of the study consisted of strain-controlled cyclic triaxial tests. All specimens were prepared to a relative density of 40% by moist tamping at 25% saturation. In order to obtain a uniform density throughout the specimen, the undercompaction method of specimen preparation suggested by Ladd (1978) was used. Following specimen preparation, the specimens were first subjected to approximately 15 minutes of CO_2 flowing through the specimen, followed by at least three pore volumes of de-aired water. The specimens were then backpressure saturated at an effective confining stress of 50 kPa and subsequently consolidated to an isotropic stress of 100 kPa. Following consolidation, the specimens were subjected to a cyclic sinusoidal axial strain at a frequency of 0.1 Hz until liquefaction occurred.

Liquefaction was defined as occurring when the pore pressure ratio, r_u, reached 0.95. For the initial effective confining stress of 100 kPa used in this study, a pore pressure ratio of 0.95 occurred when effective stress on the specimen reached 5 kPa. This criterion was chosen because it was noted that many specimens reached a pore pressure ratio of 0.95 relatively quickly, but then did not reach a pore pressure ratio of 1.0 until after a significant number of additional cycles of loading. For example, for the 50-mm diameter specimen subjected to a single amplitude strain of 0.15%, a pore pressure ratio of 0.95 was reached in 17.1 cycles, but a pore pressure ratio of 1.0 was not reached for an additional 6.0 cycles (i.e. in 23.1 cycles of loading).

4 RESULTS

The testing program consisted of 16 cyclic triaxial tests performed on four groups of specimens of different volumes. A summary of test results showing the specimen volume, applied single-amplitude axial strain, the number of cycles of loading required to cause liquefaction, the normalized dissipated energy per unit volume required to induce liquefaction and the pseudo energy capacity for each test is provided in Table 1.

The results of the testing program were examined in three ways:

- The cyclic resistance curves for the various specimen volumes (i.e. the relationship between the cyclic strain and the number of cycles of loading required to cause liquefaction);
- the normalized dissipated energy per unit volume required to induce liquefaction; and
- the pseudo energy capacity of the soil specimen.

4.1 Cyclic Resistance Curves

The most common way of using cyclic triaxial test results to analyze liquefaction resistance is through a cyclic resistance curve. To develop these curves the number of cycles of loading required to cause liquefaction in a test is plotted on the horizontal axis (typically using a logarithmic scale) and the corresponding the cyclic strain is plotted on the vertical axis. The results of several tests performed on similar specimens are plotted in this manner, a curve is fit through the data, and the cyclic strain corresponding to an appropriate number of cycles of loading is determined. This strain is then compared to the cyclic strain expected for the design earthquake.

The cyclic resistance curves for the various specimen volumes are presented in Figure 1, and the cyclic stress ratios required to produce liquefaction in 5, 15 and 25 cycles are presented in Table 2. These number of cycles to cause liquefaction were chosen to represent a small (~M5), a medium (~M7.5), and a large (~M8) earthquake respectively (Seed and Idriss, 1982; Green and Terri, 2005). As may be seen from Figure 1 and Table 2, specimen volume has little effect on the cyclic resistance curve of the soil, with all values in Table 2 falling within 15% of mean for each number of cycles.

Table 1: Summary of Test Results

Test	Cyclic Strain (%)	Cycles to Liq'n	Normalized Dissipated Energy per Unit Volume	Pseudo Energy Capacity
3.4 cm #1	0.15	13.0	0.01564	0.0185
3.4 cm #2	0.20	10.0	0.01673	0.0203
3.4 cm #3	0.25	5.9	0.01373	0.0155
3.4 cm #4	0.30	4.9	0.01564	0.01883
5.1 cm #1	0.15	11.1	0.01398	0.0151
5.1 cm #2	0.20	10.0	0.01730	0.0181
5.1 cm #3	0.25	7.0	0.01729	0.0205
5.1 cm #4	0.30	5.0	0.01561	0.0305
7.1 cm #1	0.15	13.0	0.01554	0.0152
7.1 cm #2	0.20	11.0	0.01388	0.0169
7.1 cm #3	0.25	6.0	0.01343	0.0200
7.1 cm #4	0.30	4.9	0.01560	0.0215
10.2 cm #1	0.15	10.0	0.00988	0.0126
10.2 cm #2	0.20	9.0	0.01291	0.0177
10.2 cm #3	0.25	6.9	0.01141	0.0169
10.2 cm #4	0.30	4.9	0.01287	0.0193

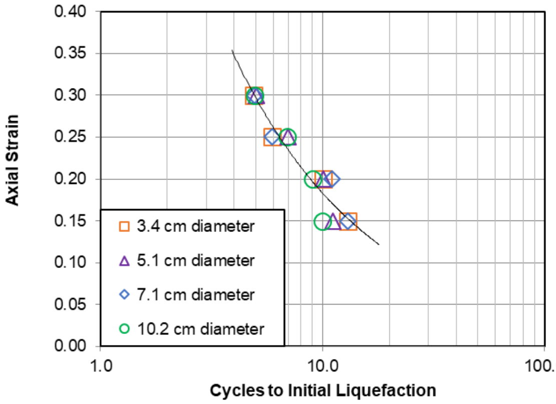

Figure 1: Cyclic resistance data for the various specimen volumes

Table 2: Cyclic Strain Required to Cause Initial Liquefaction for 5, 15, and 25 Cycles of Loading

Specimen Diameter	Single-Amplitude Strain to Cause Initial Liquefaction in 5 Cycles	Single-Amplitude Strain to Cause Initial Liquefaction in 15 Cycles	Single-Amplitude Strain to Cause Initial Liquefaction in 25 Cycles
3.4 cm	0.292	0.143	0.103
5.1 cm	0.312	0.131	0.088
7.1 cm	0.291	0.148	0.108
10.2 cm	0.311	0.116	0.074

4.2 Normalized Dissipated Energy per Unit Volume

Next, the cyclic triaxial test data were analyzed with respect to the specimen volume and the normalized dissipated energy per unit volume at the time of initial liquefaction. As mentioned above, the normalized dissipated energy per unit volume at the time of initial liquefaction for the 16 cyclic triaxial tests are listed in Table 1. The normality of the distribution of these values was checked for each specimen volume and for the combined data using normal probability plots (NIST/SEMATECH 2020). In each case the data was found to be normally distributed at the 5% significance level. To determine the significance level of the distribution, the coefficient of determination, R^2, of a best-fit line regressed through the points on the normal probability plot was compared to a critical value based on the significance level and the number of data points. Table 3 summarizes the coefficients of determination and the minimum coefficient of determination for 5% significance level for the individual specimen volumes and for the combined data. The normal probability plot for the combined data is shown in Figure 2.

Table 3: Regression Data from Normal Probability Plots for Normalized Dissipated Energy per Unit Volume

Specimen Diameter	Number of Data Points	Coefficient of Determination for the Test Data	Minimum Coefficient of Determination for 5% Significance Level
3.4 cm	4	0.9463	0.8666
5.1 cm	4	0.9366	0.8666
7.1 cm	4	0.9231	0.8666
10.2 cm	4	0.9387	0.8666
All	16	0.9717	0.9405

The mean value of normalized dissipated energy per unit volume at the time of initial liquefaction for each specimen volume and the overall mean for all tests is summarized in Table 4. Also included in the table are the standard deviations and coefficients of variation within each specimen volume and the overall standard deviation and coefficient of variation.

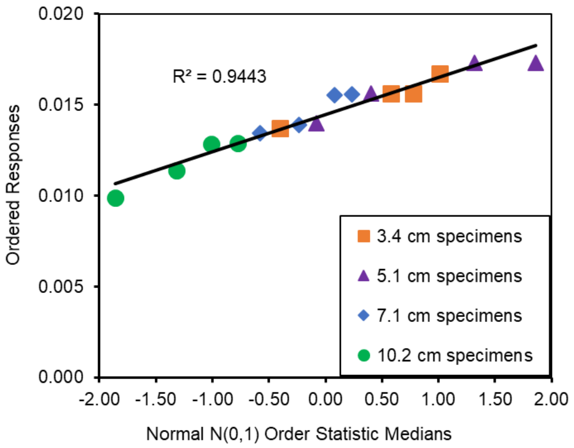

Figure 2: Normal probability plot for normalized dissipated energy per unit volume for all specimen volumes

Table 4: Normalized Dissipated Energy per Unit Volume Data by Specimen Volume

Specimen Diameter	Mean	Standard Deviation	Coefficient of Variation
3.4 cm	0.0154	0.00125	0.081
5.1 cm	0.0174	0.00162	0.099
7.1 cm	0.0146	0.00112	0.076
10.2 cm	0.0118	0.00144	0.122
All	0.0131	0.00162	0.125

In order to determine whether the means of the dissipated energy per unit volume at the time of initial liquefaction were the same for the various loadings, two-sample t-tests assuming unequal variances were performed for each of the six specimen volume pairings. The null hypothesis, H_o, was defined as the means are the same for any pair of specimen volumes

The tests indicated that the null hypothesis could not be rejected at the 5% level for the three specimen volume pairings that did not contain the 10.2 cm diameter specimens (i.e. 3.4 cm and 5.1 cm, 3.4 cm and 7.1 cm, 5.1 cm and 7.1 cm). This indicates that there is a very high likelihood that the means are the same and that the normalized dissipated energy required at the time of initial liquefaction is the same regardless of specimen volume for the 3.4 cm, 5.1 cm and 7.1 cm diameter specimens. The results of the two-sample t-tests are presented in Table 5.

The tests also indicated that the null hypothesis should be rejected at the 5% level for the three specimen volume pairings that contained the 10.2 cm diameter specimens (i.e. 3.4 cm and 10.2 cm, 5.1 cm and 10.2 cm, 7.1 cm and 10.2 cm). This indicates that there is a reasonable likelihood that the means are different and that the normalized dissipated energy required at the time of initial liquefaction is less for the 10.2 cm diameter specimens. The results of the two-sample t-tests are presented in Table 5.

The 10.2 cm diameter specimens resulted in the four lowest values of normalized dissipated energy per unit volume required to induce liquefaction. Isolating the cause for this will require further testing, however, four potential causes for the differences have been identified by the authors:

- The larger specimens actually require a lower normalized dissipated energy per unit volume to initiate liquefaction
- The four tests are a statistical anomaly and randomly produced the four lowest tests
- The larger specimen volume is less affected by the latex membrane than the other, smaller volumes tested
- The 10.2 cm diameter specimens had a lower height to diameter ratio (1.84) than the smaller diameter specimens, which all had ratios between 2.00 and 2.23

4.3 Pseudo Energy Capacity (PEC)

The initial evaluation of the pseudo energy capacity data revealed one point that clearly did not fit the remainder of the data. The outlier was from the test on the 5.1 cm specimen with a loading of 0.15% single-amplitude axial strain, which resulted in a pseudo energy capacity of 0.0305. The values for the other 15 tests ranged from 0.0126 to 0.0215, with a mean of 0.0178 and a standard deviation of 0.00245. This means the value of 0.0305 falls 5.2 standard deviations above the mean. It was therefore decided to remove the data point from the analysis. All further discussion of the pseudo energy capacity will be based upon three tests for the 5.1 cm specimens and 15 tests for the entire data set.

Table 5: The Results of the Two-Sample t-Tests of Means for the Normalized Dissipated Energy per Unit Volume

Population 1 Specimen Diameter	Population 2 Specimen Diameter	t-Stat	Critical t-values	Decision
3.4 cm	5.1 cm	-0.600	±2.447	Fail to Reject Ho
3.4 cm	7.1 cm	0.984	±2.447	Fail to Reject Ho
3.4 cm	10.2 cm	3.858	±2.447	Reject Ho
5.1 cm	7.1 cm	1.473	±2.571	Fail to Reject Ho
5.1 cm	10.2 cm	3.992	±2.447	Reject Ho
7.1 cm	10.2 cm	3.127	±2.447	Reject Ho

Following the removal of the aberrant data point, the test data showed that there is no statistically significant effect of specimen volume on the pseudo energy capacity. The pseudo energy capacity of the specimens was found to be both normally distributed and independent of specimen volume. As with the normalized dissipated energy per unit volume of soil, the normality of the distribution of values was checked for each specimen volume and for the combined data using normal probability plots (NIST/SEMATECH, 2009). In each case, the data was normally distributed at the 5% significance level. Table 6 summarizes the coefficients of determination and the critical coefficients of determination for the individual specimen volumes and for the combined data. The normal probability plot for the combined data is shown in Figure 3.

The mean value of pseudo energy capacity for each specimen volume and the overall mean for all tests is summarized in Table 7. Also included in the table are the standard deviations and coefficients of variation within each specimen volume and the overall standard deviation and coefficient of variation.

Table 6: Regression Data from Normal Probability Plots for Pseudo Energy Capacity Data

Specimen Diameter	Number of Data Points	Coefficient of Determination for the Test Data	Minimum Coefficient of Determination for 5% Significance Level
3.4 cm	4	0.9605	0.8666
5.1 cm	3	0.9463	0.8666
7.1 cm	4	0.9827	0.8666
10.2 cm	4	0.9514	0.8666
All	15	0.9855	0.9405

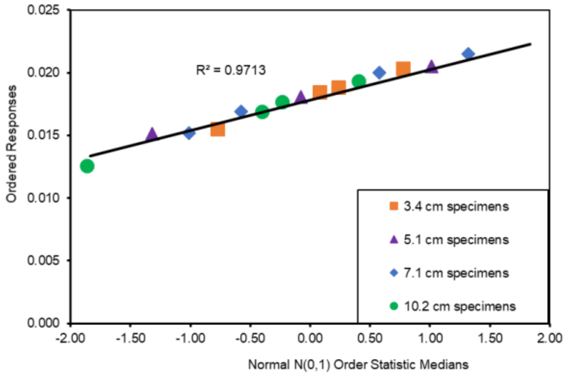

Figure 3: Normal probability plot for pseudo energy capacity for all specimen volumes

Table 7: Pseudo Energy Capacity Data by Specimen Volume

Specimen Diameter	Mean	Standard Deviation	Coefficient of Variation
3.4 cm	0.0183	0.00201	0.110
5.1 cm	0.0179	0.00269	0.150
7.1 cm	0.0184	0.00288	0.156
10.2 cm	0.0166	0.00287	0.174
All	0.0178	0.00245	0.138

In order to determine whether the means of the pseudo energy capacity were the same for the various loadings, two-sample t-tests assuming unequal variances were performed for each of the six specimen volume pairings. The null hypothesis, H_o, was defined as the means are the same for any pair of specimen volumes. The tests indicated that the null hypothesis could not be rejected at the 0.05% level for any of the specimen volume pairings. This indicates that there is a very high likelihood that the means of the pseudo energy capacity are the same. The results of the two-sample t-tests are presented in Table 8.

Table 8: The Results of the Two-Sample t-Tests of Means for the Pseudo Energy Capacity

Population 1 Specimen Diameter	Population 2 Specimen Diameter	t-value	Critical t-values	Decision
3.4 cm	5.1 cm	-0.214	±2.776	Fail to Reject H_o
3.4 cm	7.1 cm	-0.064	±2.571	Fail to Reject H_o
3.4 cm	10.2 cm	0.943	±2.571	Fail to Reject H_o
5.1 cm	7.1 cm	-0.240	±2.571	Fail to Reject H_o
5.1 cm	10.2 cm	0.594	±2.571	Fail to Reject H_o
7.1 cm	10.2 cm	0.868	±2.447	Fail to Reject H_o

5 CONCLUSIONS

A series of cyclic triaxial tests were performed to evaluate the effect of different specimen volume on the relationship between dissipated energy, liquefaction and pseudo energy capacity. Sixteen strain-controlled cyclic triaxial tests were performed on specimens of Ottawa C-109 sand These specimens were tested in groups of four with one of four specimen volumes: 1) 3.4 cm diameter (volume of 69 cm3), 2) 5.1 cm diameter (volume of 206 cm3), 3) 7.1 cm diameter (volume of 610 cm3) and 4) 10.2 cm diameter (volume of 1525 cm3).

From this study the following conclusions were drawn:

- Specimen volume has little effect on the applied strain level required to liquefy the soil in a given number of cycles. At a given number of cycles to failure, the applied strain level required to cause liquefaction fall within 15% for each level examined.
- The normalized dissipated energy per unit volume required to initiate liquefaction was found to be normally distributed both within each specimen volume and across all sixteen specimens encompassing all four specimen volumes.
- Hypothesis testing at the 5% level indicates that the normal normalized dissipated energy per unit volume required to initiate liquefaction is likely the same for the 3.1 cm diameter, 5.2 cm diameter and 7.1 cm diameter specimens.
- Hypothesis testing at the 5% level indicates that the normal normalized dissipated energy per unit volume required to initiate liquefaction is different for the 10.2 cm diameter specimens than it is for the 3.1 cm diameter, 5.2 cm diameter and 7.1 cm diameter specimens. The reason for this difference is not known. Determining the reason for this difference will require further testing, but four possible causes were identified by the authors.
- The pseudo energy capacity was normally distributed both within each specimen volume and across all sixteen specimens encompassing all specimen volumes.
- Hypothesis testing at the 5% level indicates that the pseudo energy capacity is likely the same for all specimen volumes.

6 REFERENCES

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