GeoVirtual 2020 September 14-16
Resilience and Innovation

Insights on Threshold Fines Content

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ABSTRACT
As increasing amounts of non-plastic silt are added to a sand, its classification transitions from being a sand to being a silty sand to being a sandy silt and eventually to being a silt. This transition leads to a change in the soil's behavior from sand-like to silt-like with a corresponding increase in compressibility and decrease in both shear strength and resistance to liquefaction. Numerous studies have shown that this change in behavior occurs over a relatively narrow range of silt contents. This range is referred to by several names in the literature, including threshold fines content (TFC). The threshold fines content represents the silt content at which the soil begins to transform from a sand matrix, in which the silt particles are entirely contained in the voids between the sand grains, to a silt matrix that contains isolated sand grains. Below the threshold fines content, the soil behaves essentially as a sand; above the threshold fines content, the soil behaves essentially as a silt.

While the concept and importance of the threshold fines content has been increasingly recognized over the last 20 years, several aspects of it have not been widely discussed in the literature. This paper will focus on four of these aspects: the existence of both an upper-bound and a lower-bound threshold fines content for a given soil, the range and distribution of threshold fines content upper-bound and lower-bound values for natural soils, the effect of relative density on the threshold fines content, and the behavior of soils with fines contents between the upper-bound and lower-bound threshold fines content.

ABSTRAIT
Au fur et à mesure que des quantités croissantes de limon non plastique sont ajoutées à un sable, sa classification passe du sable au sable limoneux au limon sableux et finalement au limon. Cette transition conduit à un changement du comportement du sol de sable à limon avec une augmentation correspondante de la compressibilité et une diminution à la fois de la résistance au cisaillement et de la résistance à la liquéfaction. De nombreuses études ont montré que ce changement de comportement se produit sur une plage relativement étroite de teneurs en limon. Cette gamme est désignée par plusieurs noms dans la littérature, y compris la teneur en fines seuils (TFC). Le TFC représente la teneur en limon à laquelle le sol commence à se transformer d'une matrice de sable, dans laquelle les particules de limon sont entièrement contenues dans les vides entre les grains de sable, en une matrice de limon qui contient des grains de sable isolés. Sous le TFC, le sol se comporte essentiellement comme du sable; au-dessus du TFC, le sol se comporte essentiellement comme un limon.

Alors que le concept et l'importance du TFC ont été de plus en plus reconnus au cours des 20 dernières années, plusieurs aspects de celui-ci n'ont pas été largement discutés dans la littérature. Cet article se concentrera sur quatre de ces aspects: l'existence à la fois d'une borne supérieure et d'une borne inférieure TFC pour un sol donné, la plage et la distribution des valeurs de limite supérieure et inférieure de TFC pour les sols naturels, l'effet de densité sur le TFC, et le comportement des sols avec des teneurs en fines entre le TFC supérieur et inférieur.

1 INTRODUCTION

As increasing amounts of non-plastic silt are added to a sand, the classification of the soil mixture transitions from sand to silty sand to sandy silt and eventually to silt. This transition leads to a fundamental change in the soil behavior from sand-like to silt-like, with a corresponding increase in compressibility and decrease in both shear strength and resistance to liquefaction. Numerous studies have shown that this change in behavior occurs over a relatively narrow range of silt contents. This range is referred to by several names in the literature, including "threshold fines content" (Thevanayagam et al. 2003), "limiting silt content" (Polito and Martin, 2001), "transitional fines content" (Yang et al., 2004), "critical fines content" (Kokusho, 2007) and "limiting fines content" (Hazirbaba and Rathje, 2009). For simplicity and clarity, this parameter will henceforth be referred to as the threshold fines content (TFC).

The threshold fines content represents the silt content at which the soil begins to transform from a sand matrix, with silt particles entirely contained in the voids between the sand grains, to a silt matrix that contains isolated sand grains. Below the threshold fines content, the soil behaves essentially as a sand; above the threshold fines content the soil behaves essentially as a silt. Variations in the size of the voids in the sand skeleton and in the density of the silt particles within those voids results in an upper-bound and a lower-bound for the threshold fines content. Between the upper- and lower-bounds, there is a transition zone over which the soil transforms from behaving as a sand to behaving as a silt.

To better understand how threshold fines content impacts soil behavior, this paper examines four aspects of the threshold fines concept, which previous studies have not widely discussed, namely:

- The existence of a lower- and upper-bound threshold fines content for a given soil
- The range and distribution of the threshold fines content for natural soils
- Effect of relative density on the threshold fines content
- Behavior of soils with fines contents in the transition zone between the upper- and lower-bound threshold fines content.

2 BACKGROUND

Numerous studies (e.g. Thevanayagam, 1998; Polito, 1999; Tao et al., 2004; Hazirbaba, 2005; Yang et al., 2006; and Kokusho, 2007; Polito and Sibley, 2019) have shown that the behavior of soils composed of sand and non-plastic silt is, in large part, controlled by the amount of silt in the specimen relative to some threshold value. Different researchers have defined and named this threshold value differently, but in every case, the soils with silt contents greater than the threshold value were "weaker" than the soils below the threshold value. This behavioral change is attributed to the soil fabric moving from one in which the silt particles are contained in the voids between the sand grains to one in which the sand grains are isolated in the silt matrix.

For a cohesionless soil with non-plastic fines, the threshold fines content represents the silt content at which the soil transforms from a sand matrix that contains silt particles in its voids to a silt matrix that contains isolated sand grains. Below the threshold fines content, the soil behaves essentially as a sand; above the threshold fines content, the soil behaves essentially as a silt. Figure 1 shows the conditions present when a soil is below, at and above its threshold fines content.

When studying the behavior of silty soils with various fines contents, Thevanayagam (Thevanayagam, 1998; Thevanayagam et al. 2003) identified a "threshold fines content" below which the coarse-grained material dominates the soil behavior and above which the fine-grained material dominates the soil behavior. The threshold fines content can be calculated using Equation 1 (Thevanayagam et al. 2003):

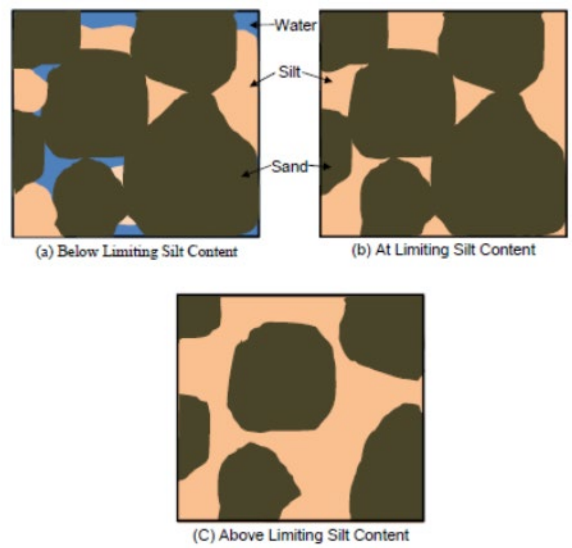

Figure 1: Schematic of a sand below, at, and above the threshold fines content.

$$\text{Threshold Fines Content} = \frac{100\mathrm{e_c}}{1+\mathrm{e_c}+\mathrm{e_{max,HF}}} = \frac{100e}{\mathrm{e_{max,HF}}} \qquad [1]$$

Where: e_c is the intergranular void ratio; $e_{Max,HF}$ is maximum void ratio of the pure silt above which it has no appreciable strength; and e is the global void ratio of the soil.

Polito (1999; Polito and Martin 2001) used the term "limiting silt content" to describe the threshold fines content and described it as the amount of silt that is present in the soil when the voids in a sand at its maximum index void ratio are completely filled with silt. This is the largest amount of silt that the sand can contain while maintaining a contiguous sand skeleton. The limiting silt content was defined as the ratio of the mass of silt to the mass of sand present at the point when the transition from a sand-dominated matrix to a silt-dominated matrix begins. The limiting silt content can be calculated for a given combination of sand and silt using Equation 2 (Polito, 1999):

$$\text{Limiting Silt Content} = \frac{\mathrm{G_{sm}e_s}}{\mathrm{G_{ss}(1+e_m)}} \qquad [2]$$

Where: G_{sm} is the specific gravity of the silt fraction; G_{ss} is the specific gravity of the sand fraction; e_s is the maximum index void ratio of the sand; and e_m is the void ratio of the silt fraction.

Hazirbaba (Hazirbaba, 2005) define "limiting fines content' as the ratio of the mass of the silt fraction present to the mass of the entire soil when the voids in a sand at its maximum index void ratio are completely filled with silt. Limiting fines content can be calculated for a given sand and silt using Equation 3 (Hazirbaba, 2005):

$$\text{Limiting Fines Content} = \frac{G_{sf}e_s}{G_{sf}e_s + G_{ss}(1+e_f)} \qquad [3]$$

Where: G_{sf} = specific gravity of the fines; G_{ss} is the specific gravity of the sand; e_f is the void ratio of the fines; is the e_s = maximum index void ratio of the sand.

Yang, Lacasse and Sandven (Yang et al., 2006) defined the "transitional fines content" as the fines content at which the voids between the sand grains are totally filled with fines. This transitional fines content represents the division between sand-like behavior and silt-like behavior and can be identified using either index text data or using the results from laboratory tests, such as the steady state line defined from large-strain undrained static triaxial tests, or cyclic resistance curves determine using cyclic triaxial tests.

In addition, the transitional fines content is a function of the specific gravities and void ratios of the sand and the silt involved. Given the appropriate index properties for the soil, the transitional fines content can be calculated using Equation 4 (Yang et al., 2006):

$$\text{Transitional Fines Content} = \frac{G_{silt}}{\left[\frac{G_{sand}(1+e_{Silt})}{e_{sand}}\right] + G_{silt}} \qquad [4]$$

Where: G_{silt} = the specific gravity of the silt; G_{sand} = the specific gravity of the sand; e_{silt} = the void ratio of the silt in the voids of the sand; e_{sand} = Void ratio of the sand.

Kokusho (Kokusho, 2007) defined the critical fines content (CF_c) for gap-graded materials as the fines content at which the fine-grained material begins to overflow the voids in the coarse-grained material. This overflowing of the voids creates an inherent change in soil structure from one that is coarse-grain supporting to one that is matrix-supporting. Critical fines content can be calculated for a given sand and silt using Equation 5 (Kokusho, 2007):

$$\text{Critical Fines Content } (CF_c) = \frac{n_c - n_c n_f}{1 - n_c n_f} \qquad [5]$$

Where: n_c = porosity of the coarse-grained fraction and $_f$ = porosity of the fine-grained fraction.

Polito and Sibley (2019) performed static and cyclic simple shear tests on specimens of sand and silt mixtures to investigate how the behavior of the soil mixtures vary with respect to the threshold fines content. From Ko-consolidated simple shear tests, they found that the friction angle of sands below the threshold fines content were 7 degrees higher than those of soils above the threshold fines content.

Polito and Sibley (2019) also used constant-volume, cyclic simple shear tests to evaluate the cyclic resistance (which they defined as the cyclic stress ratio required to trigger initial liquefaction in 15 cycles of loading) of a series of specimens of sand and silt mixtures. The specimens were prepared to the same relative density at silt contents ranging from zero (pure sand) to 45% silt. The study found that the cyclic resistance ratio was independent of silt content for mixtures below the lower-bound threshold fines content. For mixtures above the upper-bound threshold fines content, the cyclic resistance was again independent of silt content; however, the cyclic resistance ratio was less than one-half of that required to induce liquefaction in the mixtures with silt contents below the threshold fines content.

They also proposed that there is a range of possible threshold fines contents bracketed by an upper-bound threshold fine content (UBTFC) and a lower-bound threshold fine content (LBTFC). These can be calculated using Equations 6 and 7.

$$\text{UBTFC} = \frac{G_{sf}(e_{max})}{G_{sf}(e_{max}) + G_{ss}(1+e_{f,min})} \qquad [6]$$

$$\text{LBTFC} = \frac{G_{sf}(e_{min})}{G_{sf}(e_{min}) + G_{ss}(1+e_{f,max})} \qquad [7]$$

Where: G_{sf} = specific gravity of the fines; G_{ss} is the specific gravity of the sand; $e_{f,min}$ is the minimum index void ratio of the fines; $e_{f,max}$ is the maximum index void ratio of the fines; e_{max} is the maximum void ratio of the sand and e_{min} is the minimum void ratio of the sand.

3 DISCUSSION

To better understand how threshold fines content impacts soil behavior, this section will discuss four aspects of the threshold fines concept, which previous studies have not widely examined. These aspects are the existence of an upper-bound and a lower-bound threshold fines content, the range and distribution of the threshold fines contents, the effect of relative density on the threshold fines content and the behavior of soils with fines contents in the transition zone between the upper-bound and lower-bound threshold fines content.

3.1 Upper-Bound and Lower-Bound Threshold Fines Content

Because it is a function of both the density of the sand and the density of the silt, the threshold fines content can vary over a range of silt contents. Below the threshold fines content, all of the fine-grained soil particles are assumed to reside in the pores created by the sand grains, therefore the threshold fines content is a function of the void ratio of the sand skeleton as well as a function of the void ratio of the fines. To calculate the maximum upper-bound threshold fines content, the maximum index void ratio of the sand is used as the sand skeleton void ratio. Similarly, to calculate the minimum lower-bound threshold fines content, the minimum ratio void ratio of the sand is used as the sand skeleton void ratio.

If silt fills the void between sand grains, the volume of the void between the sand grains is considered equal to the volume of the silt grains and the volume of the voids between them. The mass of the silt in the void varies with its density and, thus, also with its void ratio. The mass of the silt in the void is at a minimum if the silt is at its maximum void ratio (minimum density) and is at a maximum if the silt is at its minimum void ratio (maximum density). Correspondingly, the silt content of the soil is at a minimum if the silt is at its maximum void ratio and is at a maximum if the silt is at its minimum void ratio (maximum density).

Sand skeleton void ratio is the void ratio that would exist in the soil if all silt and clay particles were removed, leaving only the sand grains to form the soil skeleton. The sand skeleton void ratio can range from the maximum to the minimum void ratio of the sand. A soil with a sand skeleton void ratio equal to its minimum void ratio and its voids filled with silt at its maximum void ratio produces the smallest possible silt content while still serving as a threshold fines content. This can be thought of as the ultimate lower-bound threshold fines content.

Conversely, a soil with a sand skeleton void ratio equal to its maximum void ratio and its voids filled with silt at its minimum void ratio produces the largest possible silt content while still serving as a threshold fines content. This can be thought of as the ultimate upper-bound threshold fines content.

Given the maximum and minimum index void ratios of the sand fraction, it is possible to calculate a sand skeleton relative density corresponding to any sand skeleton void ratio. For any sand skeleton relative density, there is a unique volume of voids and therefore a unique upper- and lower-bound threshold fines content. These upper and lower bounds are a function of the sand skeleton void ratio and the void ratio of the fines contained within the voids. For a given density of fines, as the sand skeleton relative density increases, the volume of the voids decreases and thus the UBTFC and the LBTFC decrease. Conversely, for a given sand skeleton relative density, as the density of the silt increases, the UBTFC and the LBTFC increase. These relationships are shown in Figure 2. For any void ratio of the sand skeleton and any void ratio of the fines, the UBTFC and LBTFC can be calculated using Equations 6 and 7.

3.2 The Effects of Relative Density

Figure 3 presents the upper- and lower-bound threshold fines content of C-109 sand mixed with #106 Sil-Co-Sil silt as the relative density of the sand skeleton increases from 0% to 100 % (i.e. as the voids between the sand grains become smaller). The data show that the maximum upper-bound threshold fines content, 30.7%, occurs when the sand skeleton is at its largest void ratio (its relative density is 0%) and the fine-grained material is in its densest state. Similarly, the minimum value of the lower-bound threshold fines content, 15.7%, occurs when the sand skeleton is at its smallest void ratio (its relative density is 100%) and the fine-grained material is in its loosest state.

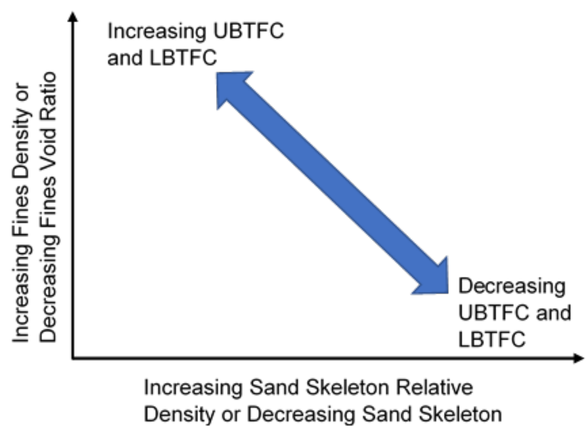

Figure 2: The effects of fines density and sand skeleton density on threshold fines content.

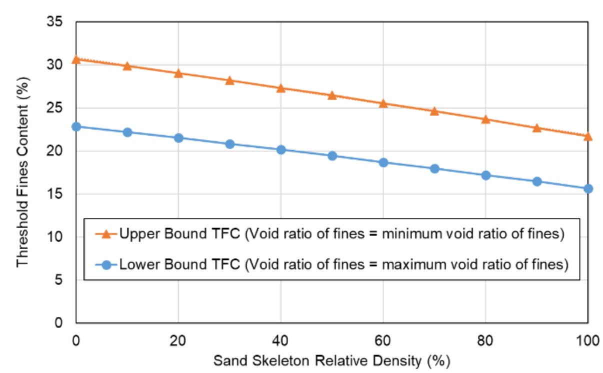

Figure 3: Threshold fines content as a function of sand fraction's relative density.

3.3 Ranges and Distributions of Upper-Bound and Lower-Bound Threshold Fines Content

Using index data reported in the literature for 62 sands and gravels and 25 silts, threshold fines contents were calculated to determine a range of typical values for maximum upper-bound and minimum lower-bound threshold fines contents. An evaluation of the 1550 combinations of the materials found that all combinations had upper-bound threshold fines contents between 18% and 48% with a mean of 34.3%, a median of 34.5% and a standard deviation of 4.7%. Similarly, it was found that all lower-bound threshold fines contents were between 8% and 37% with a mean of 18.4%, a median of 18.3% and a standard deviation of 3.7%. Figure 4 and Figure 5 provide histograms of the distribution of the maximum upper- and minimum lower-bound threshold fines contents, respectively.

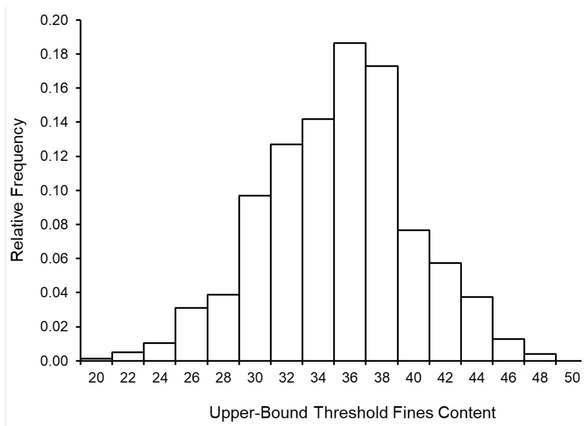

Figure 4: Distribution of upper-bound threshold fines contents.

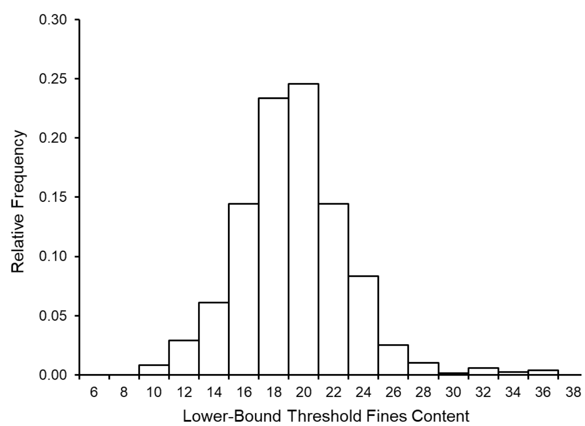

Figure 5: Distribution of lower-bound threshold fines contents.

3.4 Behavior of Soils between the Upper-Bound and Lower-Bound Threshold Fines Content

Polito and Sibley (2019) used simple shear tests to evaluate the static shear strength and the cyclic resistance of mixtures of C-109 sand mixed with #106 Sil-Co-Sil silt varying from zero to 45% silt by weight.

3.4.1 Friction Angle

When evaluating the data with respect to the upper- and lower-bound threshold fines contents, the silt content at which the friction angle begins to decrease with increasing silt content occurs between the upper- and lower-bound threshold fines contents of 21.0% and 27.4%. The friction angle continued to decrease with increasing silt content until a silt content of approximately 35% is reached. The reason for this continued decrease above the upper-bound threshold fines content is that there is still likely some localized sand grain to sand grain contact even while the volume of the silt present is greater than the volume of the voids formed by the sand grains. Additionally, even when small amounts of silt separate the sand grains, their proximity with respect to each other still influences the behavior of the soil mass. It is not until the sand grains are significantly separated by the silt particles that the soil behavior is fully controlled by the silt matrix.

3.4.2 Cyclic Resistance

Strain-controlled constant-volume cyclic simple shear tests were performed to evaluate the cyclic resistance of each of the soil mixtures. For this study, cyclic resistance was defined as being the single-amplitude shear strain required to initiate liquefaction in 15 cycles of loading. The cyclic resistance of each soil mixture was quantified from its cyclic resistance curve. The cyclic resistances were determined by fitting a best-fit line through the data and determining the shear strain level corresponding to 15 cycles of loading.

The mixtures with silt contents between 20% and 30% fall into the transition zone between the sand-dominated matrix and the silt-dominated matrix. They have cyclic resistances intermediate to the cyclic resistances of the soils above the upper bound and the soils below the lower-bound threshold fines contents. The cyclic resistance decreases with increasing silt content in this zone In these soils, both the sand grains and the silt grains contribute to the cyclic resistance: as the silt content increases, the influence of the sand grains decreases, the influence of the silt matrix increases, and the cyclic resistance decreases.

3.4.3 Normalized Dissipated Energy per Unit Volume

In addition to the more common stress-based approach (Seed and Idriss, 1971; Youd et al., 2001), liquefaction susceptibility analyses can be performed using an energy-based approach. In such an approach, the demand is quantified by the normalized dissipated energy per unit volume imparted by the earthquake and the capacity is quantified by the normalized dissipated energy per unit volume required to initiate liquefaction in the soil (NDEPUV). The demand is a function of the stress-strain behavior of the soil under the assumed seismic loading and the capacity can be determined through laboratory tests such as a cyclic simple shear test.

The normalized dissipated energy per unit volume, W_s, is the energy dissipated per unit volume of soil divided by the initial effective confining pressure. For cyclic simple shear loadings, W_s, the NDEPUV required to initiate liquefaction can be calculated by Equation 8 (Green, 2001):

$$W_s = \frac{1}{2\sigma'_o}\sum_{i=1}^{n-1}(\tau_{i+1} + \tau_i)(\gamma_{i+1} - \gamma_i) \qquad [8]$$

Where: Ws is the the normalized dissipated energy per unit volume required to initiate liquefaction in the soil; σ'_o is the initial mean effective confining stress; n is the number of load increments applied to the specimen in order to initiate liquefaction; τ_i and τ_{i+1} are the applied shear stresses at load increment i and i+1, respectively; and γ_i and γ_{i+1} are the shear strains at load increment i and i+1, respectively.

In their study, Polito and Sibley (2019) found that the normalized dissipated energy required to initiate liquefaction for soils with silt contents below the lower-bound threshold fines content was larger than for specimens above the upper-bound threshold fines content.

The shear stress and shear strain measurements obtained during the strain-controlled, constant-volume cyclic simple shear tests were used to calculate the normalized dissipated energy per unit volume required to initiate liquefaction in each specimen using Equation 6. Mixtures of Ottawa C-109 sand with up to approximately 18% silt required an average NDEPUV of 0.022 to initiate liquefaction. These mixtures represent soils with silt contents below the lower-bound threshold fines content of 21.0%. As previously discussed, the behavior of these soils is dominated by their sand fraction with little contribution from the silt fraction, which is contained almost exclusively in the voids.

Those mixtures with silt contents of 30% or greater required an average NDEPUV of approximately 0.007 to initiate liquefaction. These mixtures represent soils with silt contents above the upper-bound threshold fines content of 27.4%. As with the cyclic resistance, the behavior of these soils is dominated by their silt fraction with little contribution from the sand grains.

The mixtures with silt contents between 18% and 30% fall into the transition zone between the sand-dominated matrix and the silt-dominated matrix. They have NDEPUV to initiate liquefaction intermediate to those of the soils above the upper-bound and below the lower-bound threshold fines contents, and decrease with increasing silt content. In these soils, both the sand grains and the silt grains contribute to the cyclic resistance.

4 CONCLUSIONS

This paper has focused on four aspects of threshold fines contents. It has shown that:

- Threshold fines content, rather than being a single value, rangers between an upper bound (where the density of the sand skeleton is at a minimum and the density of the silt is at a maximum) and lower bound (where the density of the sand skeleton is at a maximum and the density of the silt is at a minimum).
- It was shown that the relative density of a soil's sand skeleton affects the threshold fines content for that soil. As sand skeleton relative density increases, both the upper and lower bound threshold fines contents decrease in a linear manner.
- The results of a study of the range and distribution of threshold fines content upper- and lower-bound values for over 1500 combinations of natural soils were presented. The upper-bound threshold fines contents were found to range from 18% to 48 % with a mean of 34.3%, a median of 34.5% and a standard deviation of 4.7%. The lower-bound threshold fines contents were found to range from 8% to 37% with a mean of 18.4%, a median of 18.3% and a standard deviation of 3.7%.
- Finally, the shear strength and cyclic behavior of soils with fines contents between the upper-bound and lower-bound threshold fines contents were found to be intermediate to those with silt contents below the lower-bound threshold fines contents and those with silt contents above the upper-bound threshold fines contents.

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