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2D and axisymmetric large strain consolidation modelling for tailings applications

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ABSTRACT
A two-dimensional large strain consolidation model is presented. The model uses a piecewise linear formulation for large strain, which implies nodes that are associated with a constant mass of solids, whose positions are updated over time. Fluxes are calculated based on 2D gradients between adjacent nodes. Regions associated with each of these nodes to calculate fluxes only deform vertically, and can slip past each other. This appears to accommodate large strains without the need for remeshing due to mesh distortion, and appears to retain sufficient accuracy. The model is validated against other analytical and numerical solutions for axisymmetric and 2D consolidation. An example analysis of 2D consolidation in a hypothetical tailings impoundment is shown. The analysis shows the formation of a beach, and how the variable water height may affect overall consolidation.

RÉSUMÉ
Un modèle bidimensionnel de consolidation à grande contrainte est présenté. Le modèle utilise une formulation linéaire par morceaux pour une grande déformation, ce qui implique des nœuds associés à une masse constante de solides, dont les positions sont mises à jour au fil du temps. Les flux sont calculés en fonction des gradients 2D entre les nœuds adjacents. Les régions associées à chacun de ces nœuds pour calculer les flux ne se déforment que verticalement et peuvent se glisser l'une sur l'autre. Cela semble s'adapter à de grandes contraintes sans avoir besoin de remaillage en raison de la distorsion du maillage, et semble conserver une précision suffisante. Le modèle est validé par rapport à d'autres solutions analytiques et numériques pour la consolidation axisymétrique et 2D. Un exemple d'analyse de la consolidation 2D dans un bassin de retenue hypothétique est présenté. L'analyse montre la formation d'une plage et comment la hauteur d'eau variable peut affecter la consolidation globale.

1 INTRODUCTION

There exist different approaches for the description of the consolidation process. The basic physical relationships in the consolidation theory were the one-dimensional Terzaghi theory which is based on the two basic assumptions including: 1) strains are small, 2) the material properties remain constant during the consolidation process (Terzaghi, 1925, Biot 1941). However, soft tailings are known to exhibit significant change in void ratios during changes in the stress state which cause large strains and change of permeability. The theory of large strain consolidation which considers the change of permeability with void ratio and the varying of void ratio with effective stress can overcome these limitations of small strain consolidation theory (Haase et al. 2000).

There are several practical applications for multi-dimensional analysis of tailings deposits. Vertical drains are used for the acceleration of consolidation process of soft tailings because using vertical drains reduced the drainage path from the thickness of a soil to half the drain spacing in the horizontal direction (Indraratna et al. 1997). Furthermore, in many soft clayey deposits, the permeability in the horizontal direction is much higher than that in the vertical direction, thus, the consolidation process can be accelerated using vertical drains (Jamiolkowski et al. 1983), or may be advantageously affected by the geometry of the impoundment.

2 2D LARGE STRAIN CONSOLIDATION MODEL (UNSATCON-2D)

Using the same framework of piece-wise linear model for CS2 (Fox and Berles 1997), Qi (2017) have developed a 1-dimensional flow–large strain consolidation formulation (in UNSATCON program). Two types of constitutive relations, two dimensional (2D) curves and three dimensional (3D) surfaces, are used to describe the hydro-mechanical behaviour of soil in the saturated and unsaturated zones, respectively. The formulation is mass conservative and smoothly model the transition between saturated and unsaturated zones (Qi 2017, Qi et al. 2019).

For 2D consolidation simulation we developed UNSATCON-2D program using the similar procedures used to calculate total stress, effective stress, pore pressure, fluid flow, and settlement to those used in CS2 (Fox and Berles 1997) and UNSATCON program (Qi 2017), but the UNSATCON-2D has been modified to account for axisymmetric geometry and two-dimensional flow. The 2D formulation used in UNSATCON-2D program is based on the following assumptions:

1) The soil is completely homogeneous, and water saturated

2) Solid particles and water are incompressible, and the deformation of the soil is completely caused by the discharge of pore water.

3) The vertical and radial flows obey Darcy's law, and the coefficient of permeability changes with the void ratio during the consolidation.

(4) All compressive strains within the soil occur in a vertical direction. The soil particles do not move along the radial and tangential directions, and no creep is considered.

(5) Horizontal sections remain horizontal during the consolidation.

(6) All vertical loads are applied instantaneously, and the load distribution is uniform over the whole cylindrical area.

Vertical total stress at each element is computed from the overburden stress and self-weight of overlaying elements but the shear stress transfer between laterally adjacent elements is not considered in the calculation of vertical total stress. The specific gravity of solids is constant for a soil layer and void ratio is assumed as constant within each element for any given time increment. Vertical effective stress σ^t_{ij} and vertical hydraulic conductivity $k^t_{v,ij}$ are computed from the void ratio $e^t_{i\ j}$ of each element using the constitutive relationships for k-e and e-σ relations. Pore pressure u^t_{ij} is the difference between total and effective stresses. Vertical fluid flow between adjacent elements is computed using the Darcy-Gersevanov law which accounts for the relative motion of fluid and solid phases (Fox et al. 2003, Qi 2017, Cao et al. 2018).

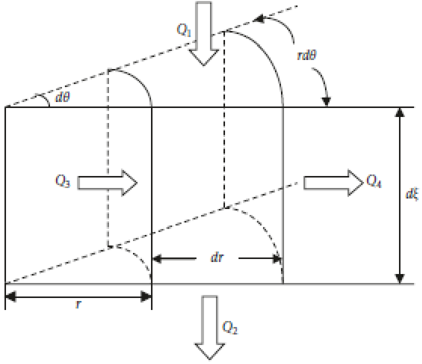


Figure 1. The diagram of coordinate system and pore water inflows and outflows (Q) for each element of model (Cao et al. 2018).

As the hydraulic conductivity of adjacent elements will generally not be equal, a harmonic average hydraulic conductivity between two neighboring elements is used to calculate the water flowing across the interface between these two elements in vertical direction (Eq. 1) and horizontal direction (Eq. 2):

$$K^t_{v,ij} = \frac{k^t_{v,ij} k^t_{v,ij-1} (H^t_{ij} + H^t_{ij-1})}{H^t_{ij} k^t_{v,ij-1} + k^t_{v,ij} H^t_{ij-1}} \qquad [1]$$

$$K^t_{h,ij} = \frac{k^t_{h,ij} k^t_{h,i-1j} (L^t_{ij} + L^t_{i-1j})}{L^t_{ij} k^t_{h,i-1j} + k^t_{h,ij} L^t_{i-1j}} \qquad [2]$$

where $k^t_{v,ij}$ is the hydraulic conductivity of each element in vertical direction and $k^t_{h,ij}$ is the hydraulic conductivity of each element in horizontal direction, H^t_{ij} is the height of each element and L^t_{ij} is the length of each element, Rj is the number of elements in vertical direction Ri is number element in horizontal direction. At the lower, upper, left, and right boundaries $K^t_{v,i1} = k^t_{v,i1}$, $K^t_{v,iRj} = k^t_{v,iRj}$, $K^t_{h,1j} = k^t_{h,1j}$, and $K^t_{h,Rij} = k^t_{h,Rij}$ respectively (Fox et al. 2003, Qi 2017, Cao et al. 2018).

The vertical flow rate ($Q^t_{v,ij}$) from element ij to element ij+1 and horizontal flow rate ($Q^t_{h,ij}$) from element ij to element i+1j (Figure 1) is calculated:

$$Q^t_{v,ij} = -K^t_{v,ij} i^t_{v,ij} \left(\frac{d\theta}{2}\right) (r^2_{i+1j} - r^2_{ij}) \qquad [3]$$

$$Q^t_{h,ij} = -K^t_{h,ij} i^t_{h,ij} r_{ij} d\theta H^t_{ij} \qquad [4]$$

where r_{ij} is the radial distance of the center of the drain to the closest side of element and thus $r_{i+1j} = r_{ij} + dr$. $i^t_{v,ij}$, $i^t_{h,ij}$ are the vertical and horizontal hydraulic gradient respectively which are:

$$i^t_{v,ij} = \frac{h^t_{ij+1} - h^t_{ij}}{z^t_{ij+1} - z^t_{ij}} \qquad [5]$$

$$i^t_{h,ij} = \frac{h^t_{i+1j} - h^t_{ij}}{r_{i+1j} - r_{ij}} \qquad [6]$$

And the total head for element ij, is:

$$h^t_{ij} = z^t_{ij} + \frac{u^t_{ij}}{\gamma_w} \qquad [7]$$

Any boundaries of the column can be specified as drained or impermeable. if any of boundaries is drained, an individual constant total head value is assigned to it. If any of boundaries is impermeable the hydraulic gradient is assumed to be zero. The smear and well resistance effects were not considered in our model at this point.

It should be mentioned that the UNSATCON-2D program uses a piecewise linear method where calculations are performed to moving nodes associated with constant mass of solids. codes with a similar formulation such as RCS1 (Fox et al. 2003) However, It UNSATCON-2D program can handle larger deformations, as regions associated with a give node are allowed to move independently of regions in adjacent columns, whereas in these other codes such regions are not permitted to slip, which may generate substantial mesh distortion. Thus, the horizontal flow can happen between any of adjacent meshes in adjacent vertical columns.

The UNSATCON-2D program is verified in this paper using 2D analytical solutions for both 2D plane strain and axisymmetric conditions.

3 ANALYTICAL SOLUTION OF 2D CONSOLIDATION

The classical axisymmetric consolidation solution incorporating vertical drains commonly assume radial drainage only, as solutions considering both vertical and radial drainage are complex. Therefore, we employed 2-D plane strain analytical solutions to evaluate both the 2D and axisymmetric versions of UNSATCON, using the permeability matching method to generate equivalent numerical predictions for the axisymmetric cases.

The governing equation of 2D-consolidation is given as a function of pore water pressure variation (Francesco 2011):

$$C_h \frac{\partial^2 u}{\partial x^2} + C_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \qquad [8]$$

where C_h and C_v are coefficient of consolidation in horizontal and vertical direction, respectively. Using the assumption of $C_h = C_v$ and with boundary conditions 0f: u(0, z, t) = 0, u(x, 0, t) = 0, u(a, z, t) = 0, u(x, b, t) = 0 Osemobor et al. 2019 proposed an analytical solution for Eq.1 using the separation of variable:

$$u(x,z,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} E_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi z}{b}\right)e^{-H\left[\left(\frac{m\pi x}{a}\right)^2+\left(\frac{n\pi z}{b}\right)^2\right]t} \qquad [9]$$

where a is width and b is height of a soil layer, $H = \frac{k}{m_v\gamma_w} = C_h = C_v$, k is hydraulic permeability and mv is coefficient of volume compressibility and γ_w is unit weight of water. Also, Emn is calculated (Osemobor et al. 2019):

$$E_{mn} = \frac{4}{ab}\int_0^b\int_0^a u_0 sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi z}{b}\right) \qquad [10]$$

4 VERIFICATION OF PROPOSED 2D MODEL

In order to verify the proposed model, two finite difference simulations were undertaken for both axisymmetric and plain strain models.

For analytical solution we used the initial conditions of a soil layer which were described in Table 1:

Table 1. Initial condition of analytical solution

Initial condition	Value
Initial height (m)	2
Length (m)	1
Top loading (kPa)	100
Coefficient of consolidation C_h, C_v (m^2/s)	3.4e-9

It should be noted that the properties of the soil layer that are used in the analytical solution and numerical simulation are from laboratory analysis carried out on soil sample obtained from Osemobor et al. 2019. Table 2 shows the initial conditions of a soil layer that are used for numerical simulation using UNSATCON-2D program.

The boundary condition for both of analytical solution and numerical simulation is the same which is permeable for top, bottom, left, and right boundaries.

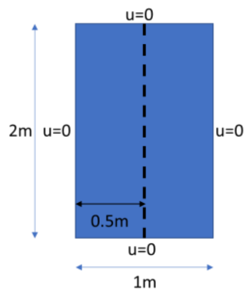
Table 2. initial condition of a soil layer used for 2D consolidation simulation

Initial condition	Value
Initial height (m)	2
Length (m)	1
Top loading (kPa)	100
Initial void ratio	2.27
Specific gravity	2.59
Coefficient of permeability k (m/s)	6.8e-11
Coefficient of consolidation C_h, C_v (m^2/s)	3.4e-9

4.1 Plane strain consolidation simulation

Comparison between the calculated Pore Water Pressure (PWP) of analytical solution and numerical simulation for a cross section at the middle of soil layer (Figure 2.a) for 100 days consolidation shows very good agreement between the results of analytical solution and simulation (Figure 2.b).

a.



b.

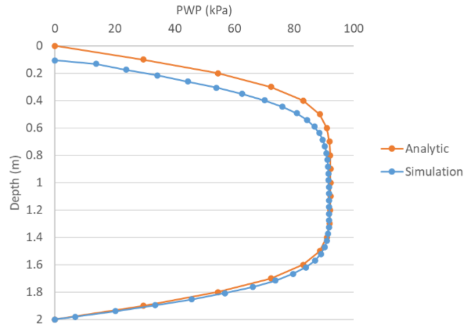


Figure 2.a. The black dash line shows the coordinate of depth profile of pore water pressure (PWP). b. Depth profile of PWP for analytical solution and simulation.

It should be mentioned which the large strain is considered in the numerical simulation. Thus, the final height of the soil layer is reduced to 1.89 m for the 100 days simulation in comparison to the height of 2 m in analytical solution. Thus, the discrepancy between the depth profiles of PWP associated with analytical solution and simulation was resulted from the large settlement that happened in the large strain consolidation simulation, which shifted down the depth of the calculated PWP of simulation by 0.11 m.

Comparison between calculated PWP of analytical solution and simulation at depth of 1 m over the width of the soil layer (Figure 3) for 100 days simulation shows very good agreement between predicted PWP by simulation and calculated PWP by analytical solution.

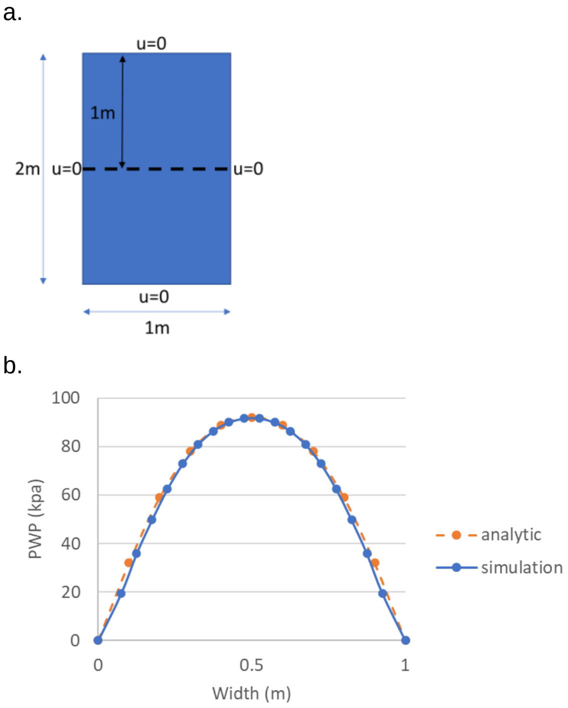


Figure 3.a. The black dash line shows the coordinate of width profile of pore water pressure (PWP). b. width profile of PWP for analytical solution and simulation.

4.2 Axisymmetric consolidation simulation

The analytical solution which was used in this paper has been calculated for plane strain analysis. Thus, the equivalence between the plane-strain and axisymmetric simulations needs to be established in order to compare the axisymmetric simulation with the analytical solution. The conversion techniques of axisymmetric solution to an equivalent plane strain model are demonstrated by several researchers. This can be achieved in three ways: a) geometric matching in which the spacing of the drains is matched while the permeability is reserved the same b) permeability matching in which the permeability coefficient is matched, while the drain spacing is kept the same. c) combination of permeability and geometric matching approach in which the plane strain permeability is computed for convenient drain spacing (Hird et al. 1992, Indraratna and Redana 1997, Indraratna and Redana 2000, Indraratna et al. 2003, Bari et al. 2011). The permeability matching technique is considered for this study. Without considering the smear and well resistance effects, the horizontal permeability for axisymmetric (k_{hax}) condition can be estimated from the used permeability in the plane strain (k_{hp}) simulation using the following transformation function (Bari et al. 2011):

$$k_{hax} = k_{hp}\left[\frac{\ln(n)-0.75}{0.67}\right] \qquad [11]$$

Further, our studies showed that the water outflow capacity and water inflow capacity of each element of model is affected by the ratio of the left interface to the right interface of a mesh. The discrepancy between the left interface to the right interface of a mesh is resulted from the radial distance of each element to the center of the drain for the axisymmetric coordinate system. Thus, to have uniform horizontal flow, the horizontal hydraulic permeability calculated from Eq. 11 for axisymmetric condition need to be modified by below equation:

$$k_h = k_{hax} * \left(\frac{average\ discharcg\ capasity\ of\ meshes}{discharcg\ capasity\ of\ each\ meshe}\right) \qquad [12]$$

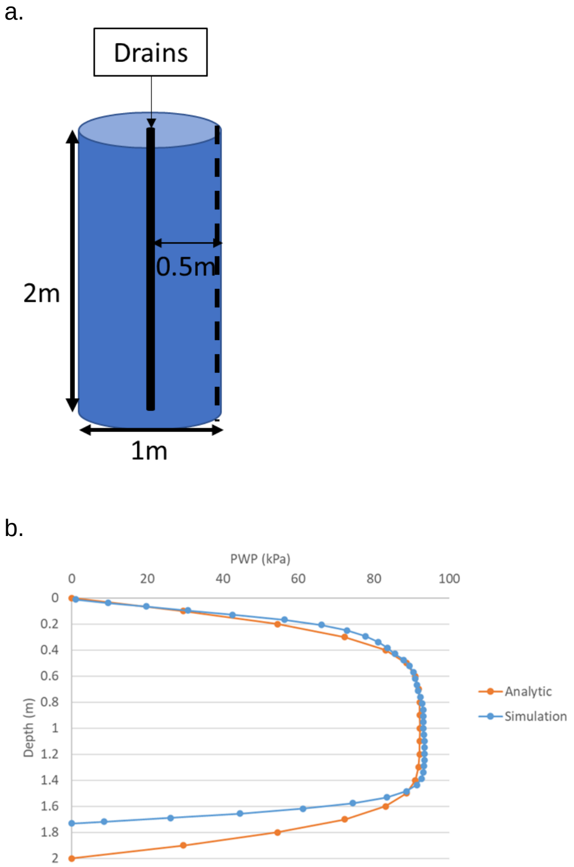


Figure 4.a. The black dash line shows the coordinate of depth profile of pore water pressure (PWP). b. depth profile of PWP for analytical solution and simulation.

To simulate a model with initial condition which is compatible with the initial condition of the analytical solution, the model of our simulation consists of a column of a soil layer with drains at the center of it. The top and bottom of column are permeable, but the surrounding of column is impermeable (Figure 4.a). There is good agreement between calculated depth profiles of PWP (Figure 4.b) of analytical solution and 2D axisymmetric simulation. Like plane strain simulation, there is a discrepancy between analytical solution and simulation in depth profile which is resulted from large settlement that happened in the consolidation simulation. Further, comparison between the width profiles of PWP (Figure 5.b) for analytical solution and 2D axisymmetric simulation shows very good agreement between predicted PWP of analytical solution with modeled PWP by 2D axisymmetric consolidation simulation. As it is clear in the Figure 5.b the PWP is equal to 0 within the drain.

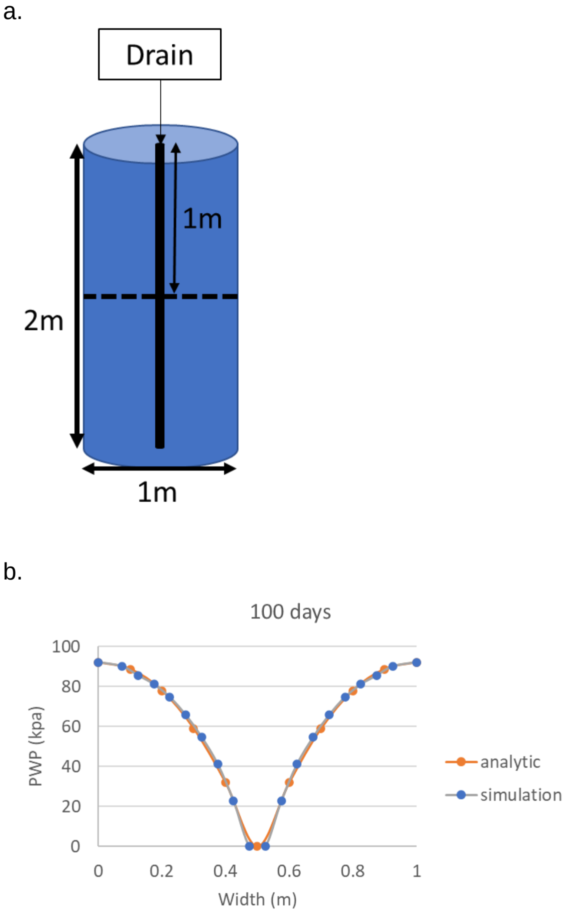


Figure 5.a. The black dash line shows the coordinate of width profile of pore water pressure (PWP). b. width profile of PWP for analytical solution and simulation.

5 HYPOTHETICAL CASE STUDY

We modelled consolidation of a hypothetical tailings pond using UNSATCON for 1D consolidation simulation and UNSATCON-2D for 2D consolidation simulation. The properties of the hypothetical tailing pond are described in Table 3. It should be added which there is decantation and no evaporation in this hypothetical tailing pond. Further, there is no vertical and bottom drains in the hypothetical tailing pond, but tailing can drain at the surface. Also, for 1D simulation we used the initial height of the middle of the hypothetical tailing pond which is equal to 48 m.

Table 3. Properties of a hypothetical tailing pond

Properties	Value
Initial height of thickest part (m)	50
Length (m)	200
Surface slop	2%
Bottom	Flat

Table 4 shows the initial conditions of a hypothetical tailing layer that are used for the 1D and 2D consolidation simulation. Equations 13 and 14 are used for prediction of the vertical and horizontal hydraulic permeability from void ratio for each time step respectively:

$$K_v=4e\text{-}11\ e^5 \qquad [13]$$

$$K_h=4e\text{-}11\ e^5 \qquad [14]$$

Where e is void ratio and k is hydraulic permeability. Equation 15 describes the relationship between effective stress (σ) and void ratio (e):

$$e = 2.8422\sigma^{-0.284} \qquad [15]$$

We used the mesh size of 0.5 m and time step of 100 second for the 1D and 2D consolidation simulations of hypothetical case study. The consolidation of hypothetical pond was simulated for 365 days. Figure 6 shows the discretized model of the hypothetical tailing pond using mesh size of 0.5 m.

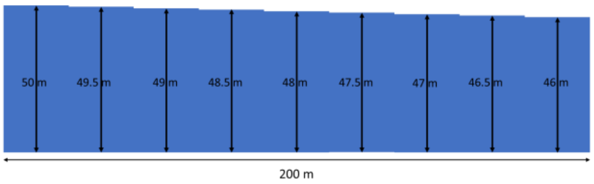


Figure 6. The discretized model of the hypothetical tailing pond using mesh size of 0.5 m.

Table 4. Initial condition of a hypothetical tailing layer

Initial condition	Value
Initial void ratio	5.6
Specific gravity	2.27

The results of 2D simulation are presented in Figure 7 for different location in the hypothetical tailing pond which is associated with different initial depth including:

50, 49, 48, 47, and 46 m. The initial height of tailing in 1D simulation is 48 m. Thus, the results of 2D simulation associated with initial height of 48 m (orange dashes line in Figure 7) can be specifically compared with the results of 1D simulation. Comparison between the variation of height versus time and the settlement versus time predicted by 1D and 2D simulations (black and orange dashed line in Figure 7.a and b) show that the 2D simulation predicts identical settlement and height relative to the 1D simulation. Consideration of the horizontal flow and the decantation of surficial drained water in the tailing pond can describe the similarity between 1D and 2D simulations. initially, the effect of horizontal flow is small in the settlement calculated by 2D simulation because the variation of lateral hydraulic head is small in the tailing pond. But, the variation of lateral hydraulic head will increase gradually. Further, decantation and the variation of tailing height in the tailing pond change the total stress from left side to the right side of the tailing pond (Figure 6). Therefore, combination of the above mentioned two phenomena lead to the prediction of the identical settlement by 2D simulation and 1D simulation. However, the discrepancy between settlement of 1D and 2D simulations will increase as the effects of horizontal flow increased in 2D simulation with increase in the variation of lateral hydraulic head in the tailing pond. Thus, the predicted final height and final settlement of 1D simulation is identical to the predicted final height and final settlement of 2D simulation on 365 day. It should be mentioned that the settlement of 1D simulation is linear within the 365 days because of the hydraulic conductivity that is used for the simulation and thus the settlement will continue till about 1000 days. But the settlement is nonlinear within the 365 days of 2D consolidation simulation because of the incorporation of horizontal flow in the 2D consolidation simulation and variation of total stress with time which resulted from the decantation of surficial water. It should be mentioned that the settlement of tailings with initial height of 47, 48, and 49 are very close to each other in Figure 7.b.

Figure 8.a shows that on the 365 day of consolidation both 1D and 2D simulations predicted almost identical void ratio for all depths in the middle of the impoundment. A small discrepancy between the predicted void ratio from the 1D simulation with the predicted void ratio of 2D simulation for shallow depths (<2 m) are related to the lack of accumulated surface drained water in the 2D simulation which causes small swelling in this part of deposit. Further, the depth profile of PWP for 1D and 2D simulations (Figure 8.b) associated with 365 day of consolidation simulations shows similar results while the PWP of 2D simulation is very slightly smaller than the PWP of 1D simulation for all depths, owing to the incorporation of horizontal flow in 2D simulation. The discrepancy between predicted PWP of 1D and 2D simulations will be larger with time.

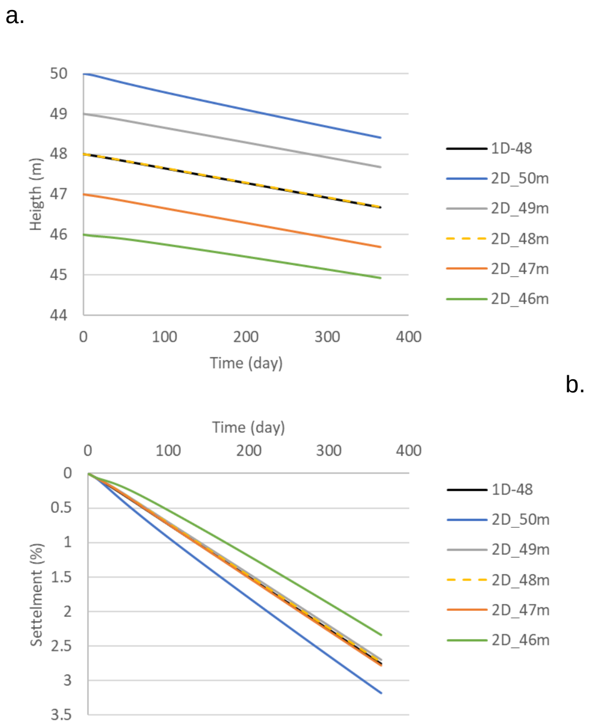


Figure 7. (a) Variation of height vs. time for 1D and 2D simulations. (b) Variation of settlement vs. time for time for 1D and 2D simulations.

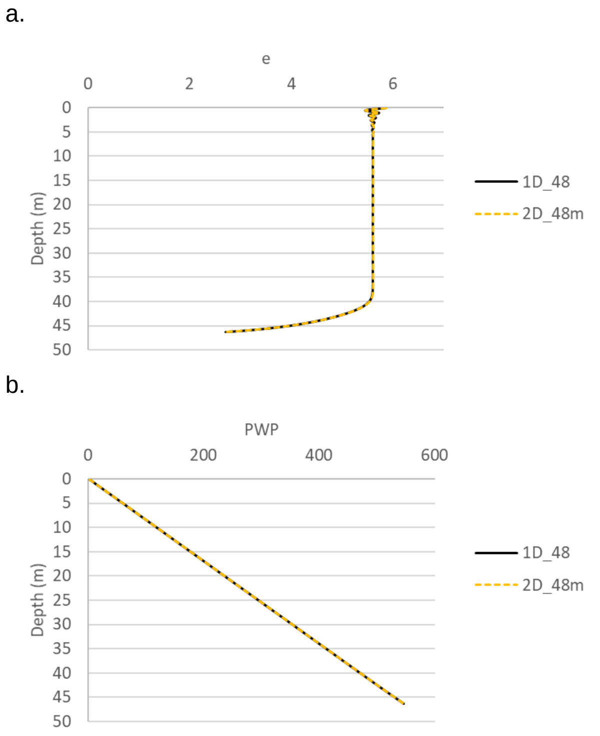


Figure 8. (a) Depth profile of void ratio for 1D and 2D simulations for 365 days consolidation. (b) Depth profile of pore water pressure for 1D and 2D simulations for 365 days consolidation simulation.

Figure 9 shows the initial and final height of tailing pond (H) and initial and final height of water (Hw) for 1D and 2D simulation. It should be noted that the final heights of water in 1D and 2D consolidation simulations is the same as final heights of tailing because of the decantation. Further, the width of tailing in the 1D simulation is assumed to be unique.

Figure 9 shows that 2D consolidation simulation predicted a reasonable and smooth height of water and tailing. Also, the tailing height predicted by 2D simulation is reasonable according to the effects of horizontal flow and lower PWP towards the initially higher (left in the figure) tailings.

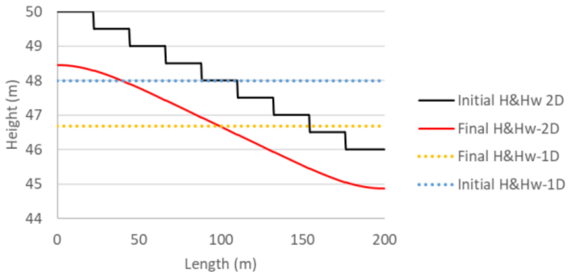


Figure 9. Initial and final height of tailing pond (H) and water (Hw) for 1D and 2D consolidation simulation of hypothetical tailing pond. (2D simulation: Initial height of tailing and water: black line, final height of tailing and water: red line; 1D simulation: Initial height of tailing and water: doted light blue line, final height of tailing and water: dotted orange line).

6 CONCLUSION

Comparison between 2D analytical model and 2D simulation for plain strain and axisymmetric conditions showed that developed UNSATCON-2D program can properly model 2D large strain consolidation.

The 1D and 2D consolidation simulations of the hypothetical tailing pond showed that accounting the horizontal flow for 2D consolidation simulation without the presence of a drain in the tailing pond will not influence significantly on the settlement in the short term but the settlement will increase in the long term. Further, the predicted depth profile of void ratio by 1D and 2D simulations for the middle of tailing pond (Initial tailing depth of 48 m) showed similar results for most of the depth of tailing deposit on 365 day of simulation. The predicted depth profile of PWP by 2D simulation shows that the PWP is very slightly smaller than the PWP predicted by 1D simulation, due to the horizontal flow in the 2D consolidation simulation. Therefore, slightly higher effective stress develops in the initially higher tailings, inducing comparatively greater settlement

7 ACKNOWLEDGEMENTS’

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8 REFERENCES

Bari, W., Shahin, M. and Nikraz, H., 2011. Axisymmetric versus Plane Strain Analyses of Ground Improvement by Prefabricated Vertical Drains Considering Soil Spatial Variability. In Proceedings of the International Conference on Advances in Geotechnical Engineering (pp. 437-444). Australian Geomechanics Society.

Biot, M.A., 1941. General theory of three-dimensional consolidation. Journal of applied physics, 12(2), pp.155-164.

Cao, Y., Yang, J., Xu, G. and Xu, J., 2018. Analysis of large-strain consolidation behavior of soil with high water content in consideration of self-weight. Advances in Civil Engineering, 2018.

Di Francesco, R., 2011. Exact solutions of two-dimensional and tri-dimensional consolidation equations. arXiv preprint arXiv:1103.6084.

Fox, P.J. and Berles, J.D., 1997. CS2: a piecewise-linear model for large strain consolidation. International Journal for Numerical and Analytical Methods in Geomechanics, 21(7), pp.453-475.

Fox, P.J., Di Nicola, M. and Quigley, D.W., 2003. Piecewise-linear model for large strain radial consolidation. Journal of Geotechnical and Geoenvironmental Engineering, 129(10), pp.940-950.

Haase, M., Exner, M. and Reichel, U., 2000. 2D Modeling of the consolidation of soft soils.

Hird, C.C., Pyrah, I.C. and Russel, D., 1992. Finite element modelling of vertical drains beneath embankments on soft ground. Geotechnique, 42(3), pp.499-511.

Indraratna, B. and Redana, I.W., 1997. Plane-strain modeling of smear effects associated with vertical drains. Journal of Geotechnical and Geoenvironmental Engineering, 123(5), pp.474-478.

Indraratna, B. and Redana, I.W., 2000. Numerical modeling of vertical drains with smear and well resistance installed in soft clay. Canadian Geotechnical Journal, 37(1), pp.132-145.

Indraratna, B., Bamunawita, C., Redana, I.W. and McIntosh, G., 2003. Modelling of prefabricated vertical drains in soft clay and evaluation of their effectiveness in practice. Proceedings of the Institution of Civil Engineers-Ground Improvement, 7(3), pp.127-137.

Osemobor, O., Nwankwo, E., Iyeke, S. and Aladenika, A., 2019. Analytical Analysis of Two-Dimensional Consolidation of Soil. American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS), 56(1), pp.218-229.

Qi, S., 2017. Numerical investigation for slope stability of expansive soils and large strain consolidation of soft soils (Doctoral dissertation, Université d'Ottawa/University of Ottawa).

Terzaghi, K.A.R.L., 1925. Principles of soil mechanics. Engineering News-Record, 95(19-27), pp.19-32.