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Considering Multiple Failure Modes in Limit Equilibrium Slope Stability Analysis: Two Methods

Shiyuan Li
Department of Mathematics – University of Waterloo, Waterloo, Ontario, Canada
Brigid Cami, Sina Javankhoshdel, Brent Corkum, Thamer Yacoub
Rocscience, Toronto, Ontario, Canada

ABSTRACT
Traditional limit equilibrium (LE) slope stability analysis methods seek to locate one single critical failure surface. However, it is often desirable to consider multiple failure surfaces when dealing with real world problems. To achieve such a goal, this paper proposes two different methods: 1) probabilistic analysis with stochastic response surface (SRS), and 2) the locally informed particle swarm with radius (LIPS-R) niching algorithm. SRS is a very fast and effective alternative to Monte Carlo or Latin Hypercube sampling for probabilistic analysis. LIPS-R is an algorithm based on a niching method called locally informed particle swarm (LIPS) which uses a radius filter and a neighbour size of two. Both methods result in multiple failure modes, albeit using very different methodologies. An embankment example is considered and results using both methods are provided in this paper. Both methods located potential failures that a deterministic analysis would not have been able to locate.

RÉSUMÉ
Les méthodes traditionnelles d'analyse de la stabilité des pentes à l'équilibre limite (LE) cherchent à localiser une seule surface de glissement critique. Cependant, il est souvent souhaitable de considérer plusieurs modes de défaillance lors du traitement de problèmes réels. Pour atteindre un tel objectif, cet article propose deux méthodes différentes: 1) l'analyse probabiliste avec la surface de réponse stochastique (SRS), et 2) l'algorithme de niching de l'essaim de particules localement informé avec rayon (LIPS-R). SRS est une alternative très rapide et efficace à l'échantillonnage Monte Carlo ou Latin Hypercube pour l'analyse probabiliste. LIPS-R est un algorithme basé sur une méthode de niching appelée essaim de particules localement informé (LIPS) qui utilise un filtre à rayon et une taille voisine de deux. Les deux méthodes entraînent de multiples modes de défaillance, mais en utilisant des méthodologies très différentes. Un exemple de remblai est considéré et les résultats utilisant les deux méthodes sont fournis dans cet article. Les deux méthodes ont permis de localiser des défaillances potentielles qu'une analyse déterministe n'aurait pas pu localiser.

1 INTRODUCTION

Traditional limit equilibrium (LE) slope stability analysis methods seek to locate the slip surface with the lowest factor of safety (FS), known as the critical failure surface. The FS value can be calculated using one of many LE methods, such as Janbu's generalized method (Janbu 1954), the Morgenstern-Price method (Morgenstern and Price 1965), or Spencer's method (Spencer 1967), amongst others.

Various methods can be used to search the slope in order to determine this most critical failure surface. These range from simple brute force methods, such as a grid search, to advanced metaheuristic methods such as Particle Swarm Optimization (PSO) (Kennedy and Eberhart 1995), or Cuckoo search (Yang & Deb 2009), amongst others.

After the global search method such as PSO has found the most critical surface, an additional local optimization method is often used to modify the geometry of the surface at a local level and thus minimize the FS of that surface further. Commonly used local optimization methods are Monte Carlo random walk (Greco 1996) and Surface Altering Optimization (SAO) (Cami et al. 2018).

However, focusing the search effort on locating the single critical failure surface has its drawbacks.

As pointed out by Reale et al. (2015) and Cho (2013) little research has been completed on slopes which could develop a number of critical slip surfaces with similar minimum FS. There are cases where the global minimum is of little practical importance, e.g. when the critical slip surface is too shallow to have any severe consequences, or when a slope is susceptible to multiple failure mechanisms, e.g. slopes with multiple benches and/or layers. Determination of ''critical'' slip surfaces is affected by the experience of the engineer or researcher, as only one failure mechanism can be identified in each trial. As noted by Griffiths et al. (2012) for slopes with multiple failure mechanisms, failure to detect some of the failure surfaces could lead to unsafe design, particularly for cases where remedial measures such as soil reinforcement are required.

As an example, one can consider a typical slope stability analysis process. The engineer collects the soil parameter and geometry data for the slope, models it in an LE slope stability analysis package, uses a robust LE method and an advanced metaheuristic search algorithm with additional local optimization to locate the most critical failure surface. By doing so, the engineer has used the most recent advancement in slope stability analysis and is confident that the critical failure surface located, with an FS of 1.10, say, is in fact the most critical. The engineer can then take the necessary steps to support this surface.

What the analysis didn't reveal, however, was that there was another surface, with an FS of 1.23 in a different location on the slope. The metaheuristic search, in seeking the global optimum, has focused its efforts in the location of the most critical surface, and not in the search for other *local* optima.

Furthermore, say the engineer used a value of 12 kPa in the second layer of the slope. But it turns out one of the data collectors had a faulty instrument that day. The true value of cohesion is 10 kPa. Running the analysis with 10 kPa results in a critical failure surface in yet another location on the slope.

In short, an engineer cannot rely on finding the single most critical failure surface and taking steps to support it. Multiple critical failure surfaces ought to be considered, both with the search algorithm itself, as well as through considering more than single fixed input parameters.

The paper considers two very different methods of considering multiple failure surfaces and demonstrates their use through an example.

The first method is probabilistic analysis, meaning an analysis that takes into account variability in the input parameters. Stochastic response surface, a method used to significantly speed up probabilistic analysis is explored in this study.

The second method is multimodal optimization. A multimodal particle swarm algorithm, LIPS-R is proposed as a means of finding multiple local minima, or critical failure surfaces.

The results of both methods are examined in a landslide model.

2 METHODS

2.1 Probabilistic Analysis with Stochastic Response Surface (SRS)

2.1.1 Probabilistic Analysis

The idea behind a probabilistic analysis is simple. Slope stability analysis requires the input of single parameter values, such as cohesion and friction angle for the materials in the slope; this is called a deterministic analysis. However, this means of doing the analysis does not consider any possibility of human or measurement error when obtaining these parameter values, nor does it consider the fact that strength parameters are not identical throughout the slope. Hence it puts too much confidence in these values.

A more rational approach would take all the estimates of cohesion, for example, and consider a range of values. If the samples are plentiful, a distribution such as normal or lognormal of cohesion can be considered. If there are only a handful, a distribution such as uniform can be considered (i.e. a range of values is considered equally); this is called a probabilistic analysis.

Once the input variables are distributions instead of single values, then each distribution can be sampled a desired number of times, and combinations of these samples are used to compute a FS for each computation, For example, if 1000 computations are desired, then each random variable is sampled 1000 times, and 1000 simulations and hence FS values are computed. The probability of failure (PF) of the slope, is defined as shown in Equation 1.

$$PF = \frac{\text{Number simulations with FS} < 1}{\text{Total number of simulations}}\ x\ 100\% \qquad [1]$$

However, computing thousands of limit equilibrium analyses takes much more time than computing a single analysis. Stochastic response surface is used to accelerate the probabilistic analysis in this study.

2.1.2 SRS

The stochastic response surface uses a small number of strategically selected computations to create a response surface of factor of safety values for various combinations of input parameters. It then predicts the factor of safety values for any combination of samples and provides an estimated probability of failure. Since a probabilistic analysis can take hours or days, this method is advantageous in significantly cutting down computation time.

The SRS methodology used in this study follows that outlined by Isukapalli (1999). The steps are outlined below:

Step 1: Convert all variable distributions to standard normal

The initial random variables are converted from the desired distribution to standard normal random variables using transformation equations (Li et al., 2011). As an example, the transformation equations for uniform, normal, and lognormal distributions are presented in Table 1.

Table 1. Transformation of three common distributions to standard normal (after Li et al., 2011)

Distribution	Probability density function, $\boldsymbol{f(x)}$	Transformation $\boldsymbol{x = f(U)}$
Uniform	$f(x) = \frac{1}{b-a}$	$x = \Phi(U)(b-a) + a$
Normal	$f(x) = \frac{1}{\sqrt{2\pi\sigma}} exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)$	$x = \mu + \sigma U$

Lognormal	$f(x)=\frac{1}{\sqrt{2\pi}\xi\sigma}exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)$	$x=exp(\xi U+\lambda)$

Step 2: Represent resulting FS in polynomial chaos expansion form.

The 3rd order Hermite polynomial expansion was used in this study, shown in Equation 2.

$$F(U_i)=a_0+\sum_{i_1=1}^{n}a_{i_1}\Gamma_1\left(U_{i_1}\right)+\sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1}a_{i_1i_2}\Gamma_2\left(U_{i_1},U_{i_2}\right)$$
$$+\sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2}a_{i_1i_2i_3}\Gamma_3\left(U_{i_1},U_{i_2},U_{i_3}\right)+\cdots$$
$$+\sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1}\cdots\sum_{i_n=1}^{i_{n-1}}a_{i_1i_2\dots i_n}\Gamma_n(U_{i_1},U_{i_2},\dots U_{i_n}) \qquad [2]$$

In the above, F is the factor of safety and U_i is the particular combination of standard normal random variables in a simulation. The coefficient vector $\boldsymbol{a}$ must be determined.

Step 3: Use a small number of computations to determine the coefficients of the polynomial in Step 2.

If n random variables are defined, the number of simulations required to be computed (N) is calculated as shown in Equation 3.

$$N = 2(1+3n+3n(n-1)/2 + n(n-1)(n-2)/6) \qquad [3]$$

These N computations are generated using Latin-Hypercube sampling (Choi et al., 2004) to ensure that the solution space is well-covered. They are then used to determine the polynomial coefficients associated with each variable or variable combination, $\boldsymbol{a}$.

Step 4: Generate Latin-Hypercube samples and plug them into the polynomial to estimate FS

In this study, the number of samples required are sampled for each random variable using Latin-Hypercube sampling. Since the coefficients, $\boldsymbol{a}$, have already been determined, the samples must simply be multiplied by $\boldsymbol{a}$, to obtain the predicted FS for each simulation. The PF is estimated from these predicted FS values.

2.2 Locally Informed Particle Swarm with Radius Filter (LIPS-R)

In 1995, a new evolutionary computation technique named particle swarm optimization (PSO) was proposed by Kennedy and Eberhard (1995). It is an algorithm inspired by the movement of organism in a bird flock and is widely used to solve unimodal optimization problems. At the beginning of PSO, n particles are generated, each with randomized position and velocity. At each iteration j, all n particles' velocity and position will be updated. In original PSO, particles' velocity and position are updated according to equation 4 and 5:

$$V^{id}=V^{id}+c1*rand()*(P_b-X)$$
$$+c2*rand()*(P_g-X) \qquad [4]$$

$$X^{new}=X+V^{id} \qquad [5]$$

Where X is the particle's current position, V is the velocity. P_b is the particle's personal best position, "pbest" and P_g is the global best position, "gbest". c1, c2 are both constants and $rand()$ is a randomly generated number in the range [0, 1].

If the newly updated particle has better position than previous personal best or global best, the information will be updated accordingly. At the end of all iterations, PSO returns the global best particle, which represents the global optimum found by the algorithm.

In 1998, Shi and Eberhard published a modified particle swarm optimizer. This modified PSO introduced the concept of inertia weight, w (Shi et al. 1998) to balance local and global search ability. In this popular variant of PSO, particles are manipulated according to equation 6 and 7:

$$V^{id}=w*V^{id}+c1*rand()*(P_b-X)$$
$$+c2*rand()*(P_g-X) \qquad [6]$$

$$X^{new}=X+V^{id} \qquad [7]$$

In the past decades, PSO has been proven to be efficient in solving unimodal problems. However, for many real-world problems, it is often desirable to find several local optima. To achieve such a goal, niching methods were introduced. Niching methods are equipped with population-based algorithms to solve multi-modal optimization problems (Li at el. 2017). Some of the famous niching PSO methods that have been developed are ring-topology-based niching PSO (Li 2010) and the Fitness-Euclidean distance Ratio (FER-PSO) (Li 2007).

In 2012, Qu proposed a distance-based locally informed particle swarm (LIPS) optimizer where instead of using the gbest and pbest, the particles are updated using information from neighbour particles "nbest." At the beginning of each iteration, the current particle's neighbours, "nbest" are calculated in terms of Euclidean distance. The velocity is then updated using information from all the nbests. This allows LIPS to eliminate the need to introduce additional niching parameters while obtaining the ability to form stable niches (Qu at el. 2012).

The proposed algorithm (LIPS-R) inherits the key concepts from LIPS with an additional radius filter. This filter allows LIPS-R to return optima (niches) that are at least some distance away from each other. At the beginning of each iteration, LIPS-R calculates the Euclidean distance between the current particle's position to each of its neighbours. The two closest neighbours' positions are N_1 and N_2. The particle's velocity is then updated using the formula given by equation 8 and 9:

$$V = w * (V + c1 * rand() * (N_1 - X) + c2 * rand() * (N_2 - X)) \quad [8]$$

$$w = \frac{2}{|2 - c - \sqrt{c^2 - 4c}|} \quad [9]$$

c1, c2 are both constants, typically set to 2.05 and c = c1+c2 (Li 2006). k is also a constant, typically set to 1.

At the end of max iterations, LIPS-R sorts all particles by its fitness value. It then filters the results by r, where r is defined by equation 10.

$$r = \sqrt{(max - min)^2} * p \quad [10]$$

Max and min mark the border of the search space and p is a user-defined parameter between [0,1]. When set to 0.1, it suggests that the user would like each optimum to be at least 10% of the search space away from others.

Once r is calculated, the radius filter works as follows: let P be the vector that stores all the sorted particles, let n be the total number of particles and let T be a vector housing only the lowest FS particle. For each particle $P[i]$, check the Euclidean distance between $P[i]$ and particles in T. If the particle's position is within distance r of particles in T, ignore it. Otherwise, push $P[i]$ to T.

LIPS-R then returns T to user, which contains all the optima that are at least r distance apart from each other.

3 EXAMPLE

The model considered in this study is shown in Figure 1. The slope consists of three layers of cohesive-frictional materials, each modeled using Mohr-Coulomb strength.

A deterministic analysis was first computed using the parameters shown in Figure 1, with Spencer's LE method. The search method used was Particle Swarm Optimization with local Surface Altering Optimization.

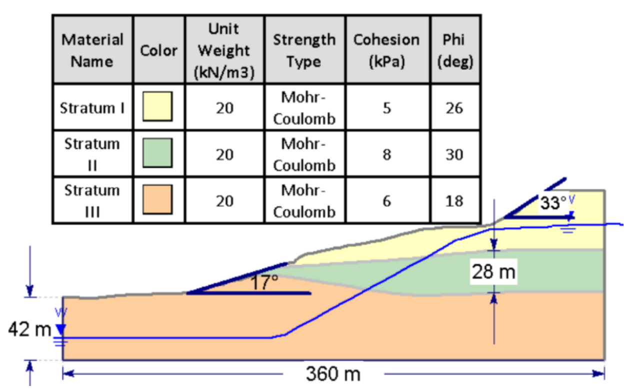


Figure 1. The geometry and parameters of the slope model considered in this study.

The results of the deterministic analysis are shown in Figure 2. The critical failure surface was found to be towards the top of the slope and resulted in an FS of 0.99.

3.1 Probabilistic Analysis with SRS

In the probabilistic analysis, each material layer was considered to have variability associated with it in the cohesion and friction angle variables. These random variables are listed in Table 2. For the first layer, for example, instead of considering a fixed cohesion of 5 kPa, a distribution of cohesion values was considered with a mean of 5 kPa and a standard deviation of 2 kPa, following a lognormal distribution.

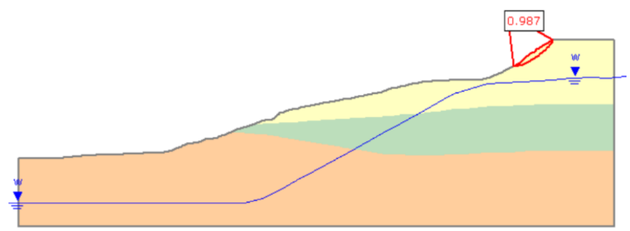


Figure 2**.** Results of deterministic analysis: FS=0.99.

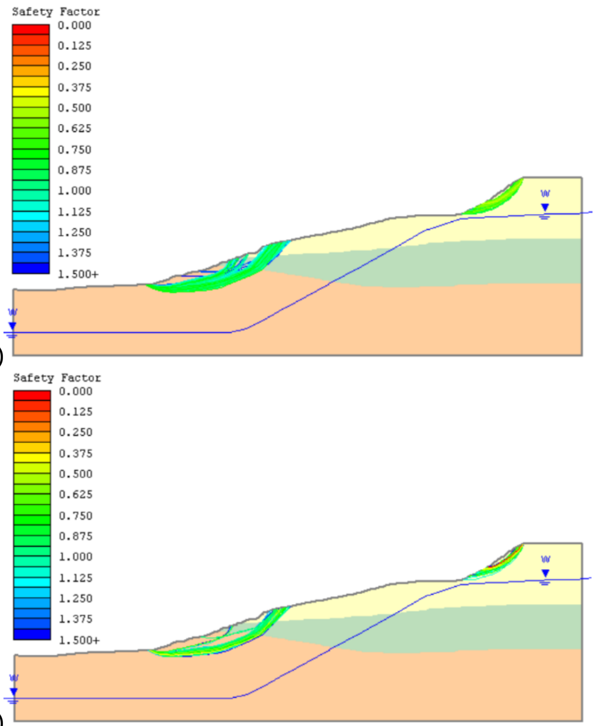
Table 2. Random variables considered in probabilistic analysis. All variables follow a lognormal distribution.

Material	Property	Mean	Standard Deviation
Stratum I	Cohesion (kPa)	5	2.0
	Friction Angle (°)	26	5.2
Stratum II	Cohesion (kPa)	8	3.2
	Friction Angle (°)	30	6.0
Stratum III	Cohesion (kPa)	6	2.4
	Friction Angle (°)	18	3.6

The number of samples computed was 5000 using Latin-Hypercube sampling, and the same LE method (Spencer) and search method (PSO with SAO) as in the deterministic case.

The results of the probabilistic analysis using both pure Latin-Hypercube (LH) sampling as well as SRS are shown in Figure 3 and Table 3.

Figure 3a shows the results of the LH analysis while Figure 3b shows that of the SRS analysis. In both parts of Figure 3, all critical surfaces found in each simulation of the probabilistic analysis are shown on the slope. It can be seen that in addition to the critical deterministic failure surface, there is another region where many critical surfaces were found in various simulations, with FS values primarily below 1.0 (see legend in Figure 3). Per Equation 3, although 5000 FS values were computed with SRS, only 168 actual analyses were computed to train the model and predict all the FS values. As such only these surfaces are displayed in Figure 3b. It is notable that the critical regions found by LH were also found by SRS.



a)

b)

Figure 3. Results of Probabilistic Analysis using a) Latin-Hypercube sampling, and b) Stochastic Response Surface.

In Table 3, the quantitative results of two methods are summarized. It can be seen that the PF and mean FS estimated by SRS are in very good agreement with that of LH. It is certainly of note however that the computation time of SRS (approximately 6 minutes) was about 4% of the LH computation time.

Table 3. Results of Probabilistic Analysis using Latin-Hypercube sampling, and Stochastic Response Surface with 5,000 simulations.

	LH	SRS
PF (%)	50.5	48.8
Mean FS	1.03	1.01
Computation time (min)	159.6	5.7

3.2 LIPS-R

LIPS-R with SAO were used to compute the results shown in Figure 4. By using LIPS-R, the search is not focusing its energies on finding the global minimum and has hence determined three different failure modes.

In addition to the FS=0.99 critical failure surface, this method also located two other critical surfaces with FS values of 1.35 and 1.51. It is interesting to note that the FS=1.35 surface appears to be in the same region as the surface located by the probabilistic analysis, even though the means of arriving at these two regions were entirely different. The third critical surface was not located by the probabilistic analysis and introduces a new failure mode to the model.

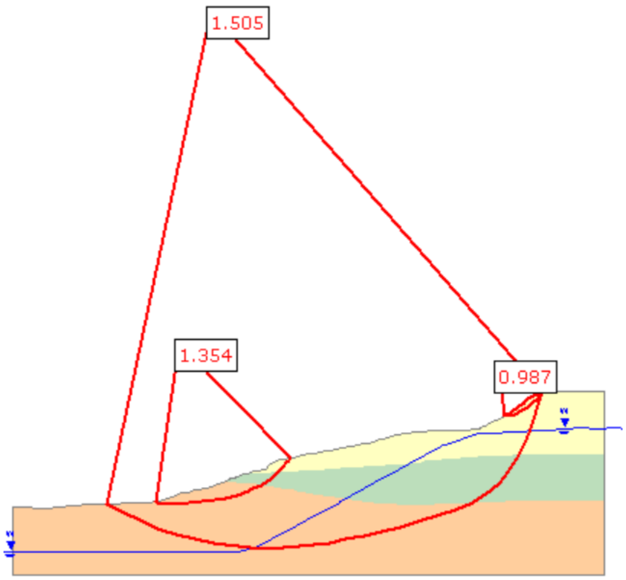


Figure 4. Results of LIPS-R multi-modal search.

4 CONCLUSION

Traditional limit equilibrium slope stability analysis methods seek to locate one single critical failure surface. However, when dealing with real world problems, it is often not sufficient to take into account the single worst case. This paper has proposed two different methods for obtaining more than the singular deterministic critical failure surface: probabilistic analysis with stochastic response surface (SRS), and locally informed particle swarm with radius (LIPS-R). SRS achieves this by taking into account the variability of input parameters. LIPS-R achieves it through an algorithm that performs local optimization instead of global optimization.

In addition to the critical deterministic surface, both methods located a second failure mode that a deterministic analysis would not have been able to locate. The LIPS-R algorithm additionally located a third failure mode not found by either the deterministic or probabilistic analyses.

In conclusion, it is not sufficient to account for a single critical failure surface in LE slope stability analysis. This study has shown that using probabilistic analysis or using niching can provide the engineer with critical information about potential failure modes that would not be found with a simple deterministic analysis.

5 REFERENCES

Bird S. and Li X. 2006. Adaptively choosing niching parameters in a PSO, Proceedings of the 8th annual conference on Genetic and evolutionary computation (GECCO '06), New York, NY, USA, 3-10

Choi, S. K., Grandhi, R. V., Canfield, R. A., & Pettit, C. L. 2004. Polynomial chaos expansion with latin hypercube sampling for estimating response variability. AIAA journal, 42(6), 1191-1198.

Cami, B., Javankhoshdel, S., Yacoub, T. and Bathurst, R.J., 2018, November. 2D Spatial Variability Analysis of Sugar Creek Embankment: Comparative Study. In International Congress and Exhibition" Sustainable Civil Infrastructures: Innovative Infrastructure Geotechnology" (pp. 118-125). Springer, Cham.

Cho, S.E., 2013. First-order reliability analysis of slope considering multiple failure modes. Engineering geology, 154, pp.98-105.

Greco, V.R. 1996. Efficient Monte Carlo Technique for Locating Critical Slip Surface. Journal of Geotechnical Engineering, 122(7): 517-525.

Griffiths DV, Huang JS, Fenton GA. Risk assessment in geotechnical engineering: stability analysis of highly variable soils. In: Geotechnical engineering state of the art and practice, keynote lectures from geocongress see2012; 2012. p. 78–101.

Isukapalli, S. S. 1999. Uncertainty analysis of transport-transformation models.

Janbu, N. 1954. Application of composite slip surface for stability analysis. In Proceedings of European Conference on Stability of Earth Slopes, Sweden, 1954 (Vol. 3, pp. 43-49).

Kennedy, J. and Eberhart, R. 1995. Particle swarm optimization, Proceedings of ICNN'95 - International Conference on Neural Networks, IEEE, Perth, WA, Australia, 4: 1942-1948

Li, D., Chen, Y., Lu, W., & Zhou, C. 2011. Stochastic response surface method for reliability analysis of rock slopes involving correlated non-normal variables. Computers and Geotechnics, 38(1), 58-68.

Li, X. 2007a. Niching Without Niching Parameters: Particle Swarm Optimization Using a Ring Topology, IEEE Transactions on Evolutionary Computation, 14(1): 150-169

Li X. 2007b. A multimodal particle swarm optimizer based on fitness Euclidean-distance ratio. Proceedings of the 9th annual conference on Genetic and evolutionary computation (GECCO '07), New York, NY, USA, 78–85

Li, X. Epitropakis, G. M. Deb, K. and Engelbrecht, A. 2017. Seeking Multiple Solutions: An Updated Survey on Niching Methods and Their Applications, IEEE Transactions on Evolutionary Computation, 22(4): 518-538

Morgenstern, N. U., and Price, V. E. 1965. The analysis of the stability of general slip surfaces. Geotechnique, 15(1), 79-93.

Qu B. Y., Suganthan P. N. and Das S. 2013. Distance-Based Locally Informed Particle Swarm Model for Multimodal Optimization, IEEE Transactions on Evolutionary Computation, 17(3): 387-402

Reale, C., Xue, J., Pan, Z. and Gavin, K., 2015. Deterministic and probabilistic multi-modal analysis of slope stability. Computers and Geotechnics, 66, pp.172-179.

Shi, Y. and Eberhart, R. 1998. A modified particle swarm optimizer, 1998 IEEE International Conference on Evolutionary Computation Proceedings. IEEE World Congress on Computational Intelligence (Cat. No.98TH8360), Anchorage, AK, USA, 69-73

Spencer, E. 1967. A method of analysis of the stability of embankments assuming parallel inter-slice forces. Geotechnique, 17(1), 11-26.

Yang, X.S. and Deb, S., 2009, December. Cuckoo search via Lévy flights. In 2009 World congress on nature & biologically inspired computing (NaBIC), IEEE, 210-214